

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

### 8-3 Rational Functions and Their Graphs

#### Standards

**A2. F.IF.A.1** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

**A2. F.IF.B.3** Graph functions expressed symbolically and show key features of the graph, by hand and using technology

#### Key Concepts

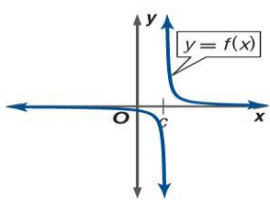
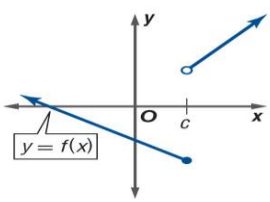
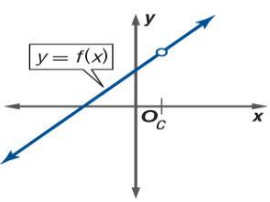
\_\_\_\_\_ – a function that you can write in the form  $f(x) = \frac{P(x)}{Q(x)}$ , where  $Q(x) \neq 0$

\_\_\_\_\_ – a graph that has no breaks, jumps, or holes.

\_\_\_\_\_ – a graph that has jumps, breaks or holes.

\_\_\_\_\_ – the point at which the graph is not continuous ( $x = a$ )

\_\_\_\_\_ – the graph has a vertical asymptote at  $x = a$  if it has non-removable discontinuity at  $x = a$ .

KeyConcept Types of Discontinuity		
<p>A function has an <b>infinite discontinuity</b> at <math>x = c</math> if the function value increases or decreases indefinitely as <math>x</math> approaches <math>c</math> from the left and right.</p> <p>Example</p> 	<p>A function has a <b>jump discontinuity</b> at <math>x = c</math> if the limits of the function as <math>x</math> approaches <math>c</math> from the left and right exist but have two distinct values.</p> <p>Example</p> 	<p>A function has a <b>removable discontinuity</b> if the function is continuous everywhere except for a hole at <math>x = c</math>.</p> <p>Example</p> 

\_\_\_\_\_ - to find a horizontal asymptote, compare the degree of the numerator to the degree of the denominator.

- If degree of numerator < degree of denominator, then the horizontal asymptote is  $y = 0$
- If degree of numerator = degree of denominator, then the horizontal asymptote is  $y =$  ratio of leading coefficients.
- If degree of numerator > degree of denominator, then there is no horizontal asymptote.

## Examples

1. (I do) Consider the rational function  $y = \frac{x+4}{x^2-x-12}$ 
  - a. What is the domain of the rational function?
  - b. Identify the points of discontinuity. Are the points of discontinuity removable or non-removable?
  - c. What are the  $x$ - and  $y$ - intercepts?
  
2. (We do) Consider the rational function  $y = \frac{2x}{x^2+12}$ 
  - a. What is the domain of the rational function?
  - b. Identify the points of discontinuity. Are the points of discontinuity removable or non-removable?
  - c. What are the  $x$ - and  $y$ - intercepts?
  
3. (They do) Consider the rational function  $y = \frac{x^2-4}{x+2}$ 
  - a. What is the domain of the rational function?
  - b. Identify the points of discontinuity. Are the points of discontinuity removable or non-removable?
  - c. What are the  $x$ - and  $y$ - intercepts?

4. (I do) What are the vertical asymptotes for the graph?

a.  $y = \frac{(x+3)}{(x-3)(x+2)}$

b.  $y = \frac{(x+7)}{(x^2+9x+14)}$

5. (We do) What are the horizontal asymptotes for the graph?

a.  $y = \frac{-4x+3}{2x+1}$

b.  $y = \frac{x-2}{x^2-2x-3}$

c.  $y = \frac{x^2}{4x-1}$

6. (They do) Graph the rational function  $y = \frac{x+1}{x^2-x-6}$

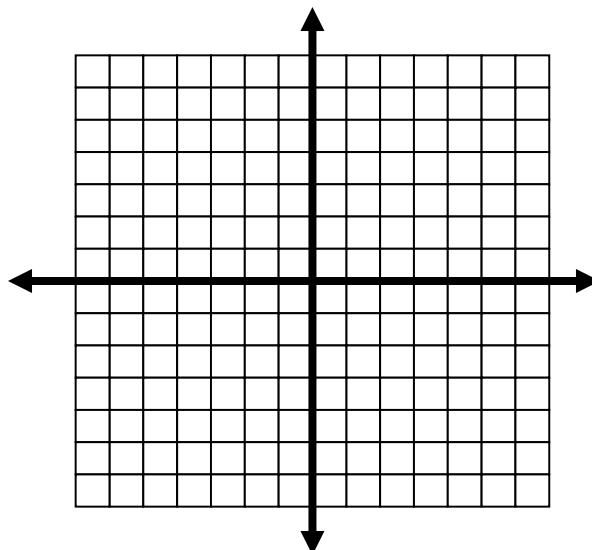
Step 1: Find HA

Step 2: Factor

Step 3: Find VA

Step 4: Find  $x$ - and  $y$ - intercepts

Step 5: Graph and get additional points on the graph



7. (They do) You work at a pharmacy that mixes different concentrations of saline. The pharmacy has a supply of two different concentrations, 0.5% and 2%. The function  $y = \frac{100(0.02)+x(0.005)}{100+x}$  gives the concentration of the mixture after adding  $x$  milliliters of the 0.5% solution to 100 milliliters of the 2%. How many milliliters of the 0.5% solution must you add for the combined solution to have a concentration of 0.9%?

**(You do) Practice 8-3: Complete your assignment on a separate sheet of paper. Show all work.**

1. State the domain, find any points of discontinuity for each rational function, state the  $x$ - and  $y$ -intercepts. Are there any vertical asymptotes? Are the points of discontinuity removable or non-removable?

a.  $y = \frac{x+5}{x^2+9x+20}$

b.  $y = \frac{x-1}{(x+1)^2}$

c.  $y = \frac{x^2-x-2}{3x^2-7x+2}$

2. Find the horizontal asymptotes.

a.  $y = \frac{x-3}{x+5}$

b.  $y = \frac{x-3}{x^2+5x+6}$

c.  $y = \frac{x^2-1}{2x+2}$

3. Sketch the graph of each rational function  $y = \frac{x+3}{x^2-7x+6}$

4. (See example 7, use the same function) How many milliliters of the 0.5% solution must be added to the 2% solution to get a 0.65% solution?