13-1 Exploring Periodic Data

State Standards

A2. F.LE.A.2 (formerly F-TF.A.2) Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

Objectives

The students will identify cycles, periods and amplitude in periodic functions.

Key Concepts

______________________________________- a function that repeats a pattern of *y*-values at regular intervals.

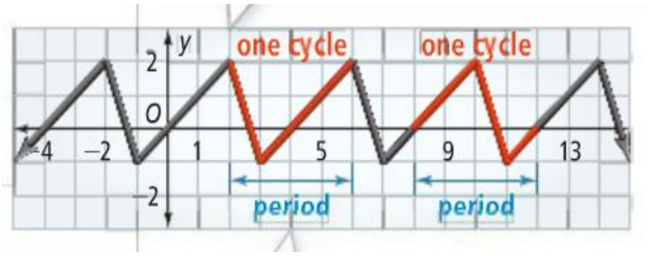
___________________- one complete pattern

___________________- the horizontal length of one cycle

_________________________- half the difference of the maximum and minimum values

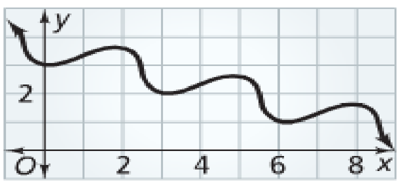
Examples

1. (I do) Identify the cycle in the periodic function. State the period.

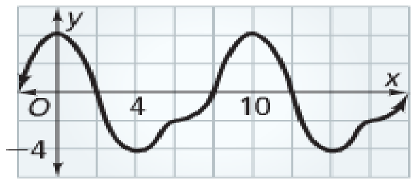


2. (We do) Is the function periodic? If so, state the period.

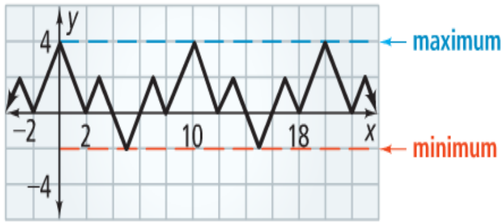
A



B



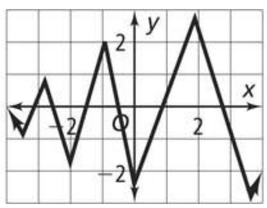
3. (They do) Find the amplitude of the periodic function.



You do Practice 13-1: Complete your assignment on a separate sheet of paper. Show all work.

1. Determine if the function is periodic. If so, find the period.

a



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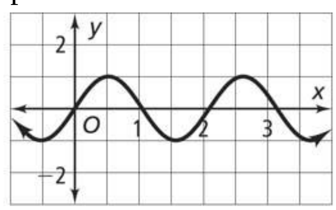
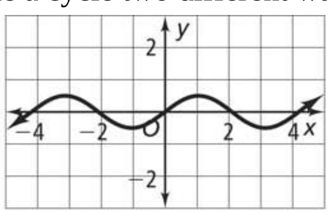


2. Name a cycle two different ways. Then determine the period and amplitude.

a



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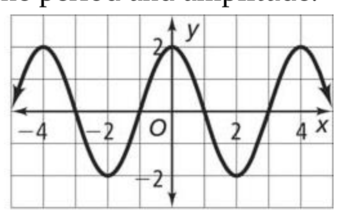


3. Sketch the graph of a periodic function with period 8 and amplitude 3. State the maximum and minimum of your function.

13-2 Angles and the Unit Circle

State Standards

A2. F.LE.A.1 (formerly F-TF.A.1) Understand and use radian measure of an angle. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. Use the unit circle to find sin θ, cos θ, and tan θ when θ is a commonly recognized angle between 0 and 2π.

Objective
The students will work with angles in standard position and find the coordinates of points on the unit circle.

Key Concepts

______________________________________- when the vertex is at the origin and one ray is on the positive *x*-axis.

____________________________________- the ray on the *x*-axis of an angle in standard position.

____________________________________-the other ray of an angle in standard form.

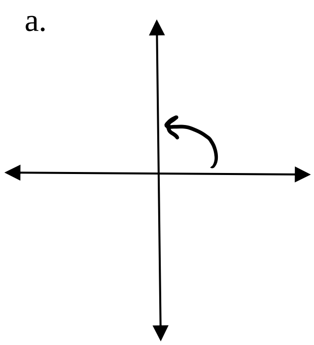
____________________________________- 2 angles in standard position when they have the same terminal side.

____________________________________- has a radius of one and its center at the origin. Points on the circle are related to periodic functions.

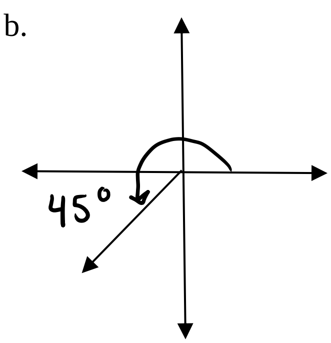
Examples

1. (I do) What is the measure of each angle?

a.

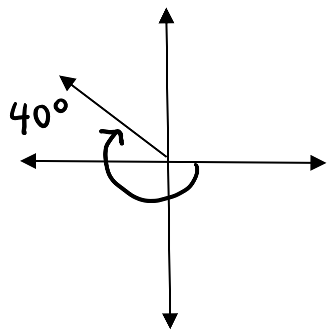


b.

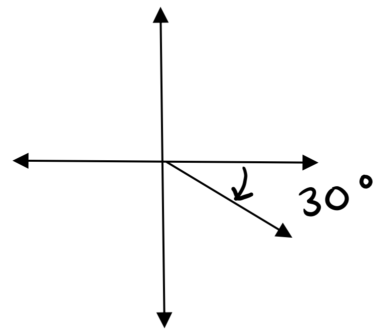


2. (We do) What is the measure of each angle?

a.



b.



3. (We do) Sketch each angle in standard position.

a. 35°

b. -320°

c. 315°

4. (We do) Identify a coterminal angle for the following.

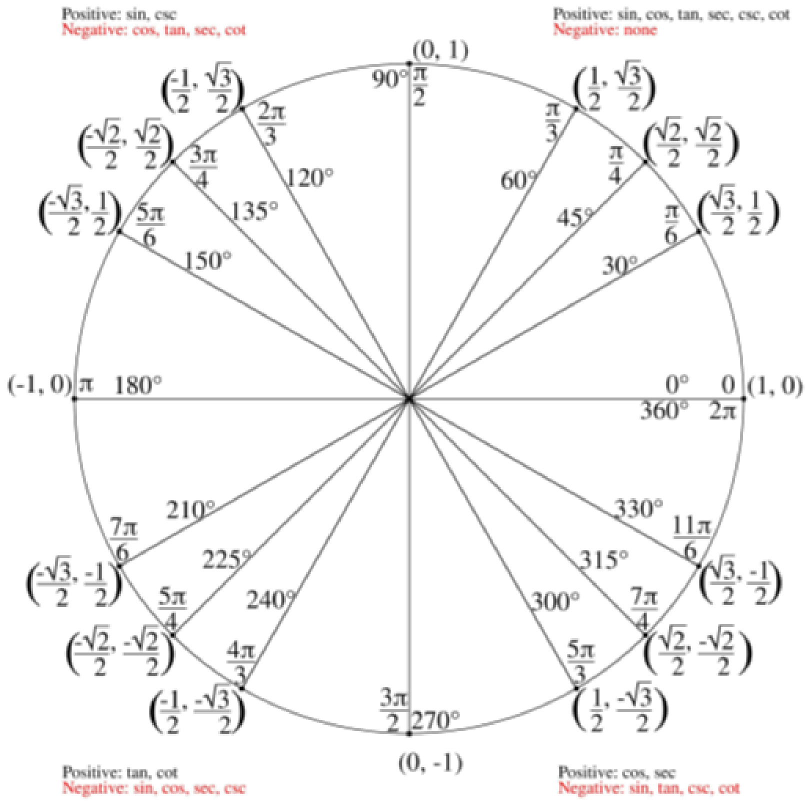
a. 300°

b. -225°

c. 90°

5. (They do) Use the unit circle to determine $\cos\theta$ and $\sin\theta$ for $\theta = 30, 60, 90, 180$ & 270

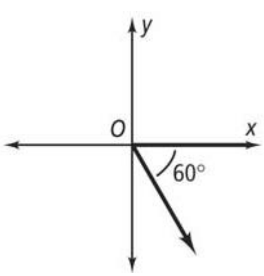
The Unit Circle



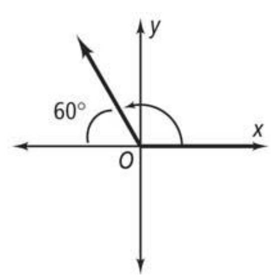
You do Practice 13-2: Complete your assignment on a separate sheet of paper. Show all work.

1. Find the measure of each angle in standard form.

a.



b.



2. Sketch the angle in standard position.

a. 100° b. 210° c. -45°

3. Identify a coterminal angle for the following.

a. -100° b. 409° c. -145°

4. Use the unit circle to determine cos 135 and sin 300.

13-3 Radian Measure

State Standards

A2. F.LE.A.1 (formerly F-TF.A.1) Understand and use radian measure of an angle. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. Use the unit circle to find sin θ, cos θ, and tan θ when θ is a commonly recognized angle between 0 and 2π.

Objective
The students will use radian measure for angles.

Key Concepts

______________________________ - an angle with the vertex at the center of the circle.

______________________________ - the portion of the circle with endpoints on the sides of the central angle and remaining points within the interior of the angle.

____________________________-the measure of a central angle that intercepts an arc.

Key Concept

take note

Key Concept **Converting Between Radians and Degrees**

To convert degrees to radians, multiply by $\frac{\pi \text{ radians}}{180^\circ}$.

To convert radians to degrees, multiply by $\frac{180^\circ}{\pi \text{ radians}}$.

Examples

1. (I do) Convert radians to degrees.
   a. $\frac{3\pi}{4}$
   b. $\frac{\pi}{2}$
   c. $\frac{5\pi}{6}$

2. (I do) Convert degrees to radians. Express your answer in terms of π and as a decimal rounded to the nearest hundredth.
   a. 27°

   b. 225°

   c. -150°

3. (We do) What are the exact values of the following?
   a. $\cos\frac{\pi}{4}$

   b. $\sin-\frac{\pi}{4}$

   c. $\cos\frac{\pi}{2}$

   d. $\sin\frac{\pi}{3}$

   e. $\cos 3\pi$

4. (They do) Calculate a coterminal angle for each of the following.

   a. $\frac{9\pi}{4}$ b. 3π

You do Practice 13-3: Complete your assignment on a separate sheet of paper. Show all work.

1. Convert radians to degrees. Round to the nearest degree.

a. $\frac{7\pi}{6}$ b. $-\frac{\pi}{6}$ c. 1.8 radians

2. Write each measure in radians. Express your answer in terms of π and as a decimal round to the nearest hundredth.

a. -45° b. 120° c. 270°

3. Calculate the exact values of the following.

a. $\sin\frac{3\pi}{4}$ b. $\cos\frac{3\pi}{2}$ c. $\cos\frac{7\pi}{6}$ d. $\cos-\frac{\pi}{4}$

4. Calculate a coterminal angle for each of the following.

a. $\frac{11\pi}{4}$ b. $-\frac{\pi}{4}$

13-8 Trigonometric Functions

State Standards

A2. F.LE.A.1 (formerly F-TF.A.1) Understand and use radian measure of an angle. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. Use the unit circle to find sin θ, cos θ, and tan θ when θ is a commonly recognized angle between 0 and 2π.
A2. F.LE.A.2 (formerly F-TF.A.2) Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

Objective
The students will evaluate and graph reciprocal trigonometric functions.

Key Concepts

Trig Ratios on Unit circle

$(cosine)\ \ cos\ \theta = x \qquad (sine)\ \ sin\ \theta = y \qquad (tangent)\ tan\ \theta = \frac{y}{x}$

$(secant)\ \ sec\ \theta = \frac{1}{x} \qquad (cosecant)\ \ \csc\theta = \frac{1}{y} \qquad (cotangent)\ \ \cot\theta = \frac{x}{y}$

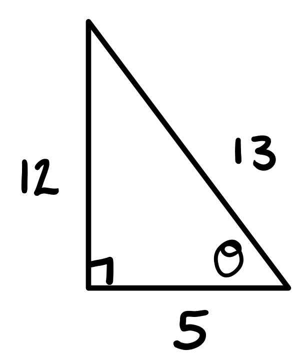
Trig Ratios with Right Triangles

$cos\ \theta = \frac{adjacent}{hypotenuse} \qquad sin\ \theta = \frac{opposite}{hypotenuse} \qquad tan\ \theta = \frac{opposite}{adjacent}$

$sec\ \theta = \frac{hypotenuse}{adjacent} \qquad \csc\theta = \frac{hypotenuse}{opposite} \qquad \cot\theta = \frac{adjacent}{opposite}$

Examples

1. (I do) Use the triangle to calculate the 6 trig functions.



2. (We do) Calculate the exact measures using the unit circle.

a. $\csc \frac{\pi}{6}$ b. $\cot \frac{-5\pi}{6}$ c. $\sec 3\pi$

3. (They do) Calculate the value. Round to the nearest thousandth.

a. $\sec 2$ b. $\cot 10$ c. $\csc 35°$

d. $\cot \frac{\pi}{4}$ e. $\sin 150°$ f. $\cos 50°$

g. $\cot \frac{\pi}{2}$

You do Practice 13-8: Complete your assignment on a separate sheet of paper. Show all work.

Find each value without using the unit circle. If the expression is undefined, write *undefined*.

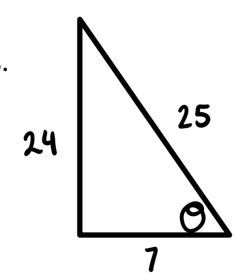
1. $\csc(-\pi)$ **2.** $\cot \frac{2\pi}{3}$ **3.** $\sec\left(-\frac{11\pi}{6}\right)$ **4.** $\csc \frac{3\pi}{4}$

5. $\cot\left(-\frac{\pi}{2}\right)$ **6.** $\csc 540°$ **7.** $\sec \frac{\pi}{3}$ **8.** $\cot\left(-\frac{\pi}{6}\right)$

Use a calculator to find each value. Round your answers to the nearest thousandth.

9. $\cot 42°$ **10.** $\csc \frac{\pi}{6}$ **11.** $\csc(-2)$ **12.** $\sec 155°$

13. Use the triangle to calculate the 6 trigonometric functions.



14-1 Trigonometric Identities

State Standards

A2.F.TF.B.3 Know and use trigonometric identities to find values of trig functions. a. Given a point on a circle centered at the origin, recognize and use the right triangle ratio definitions of sin θ, cos θ, and tan θ to evaluate the trigonometric functions. b. Given the quadrant of the angle, use the identity $\sin^2 \theta + \cos^2 \theta = 1$ to find sin θ given cos θ, or vice versa.

Objective
The students will verify identities using basic identities.

Key Concepts

______________________________________ a trigonometric equation that is true for all values of the variable for which all expressions in the equation are defined.

Examples

1. Verify the identity.
   a. $\sin\theta\,(\sec\theta) = \tan\theta$
   b. $\frac{\csc\theta}{\sec\theta} = \cot\theta$
   c. $1 + tan^2\,\theta = \ sec^2\theta$
   d. $tan^2\theta - sin^2\theta = tan^2\theta sin^2\theta$

You do Practice 14-1: Complete your assignment on a separate sheet of paper. Show all work.

Lesson Check

Do you know HOW?

Verify each identity.

1. $\tan\theta \csc\theta = \sec\theta$
2. $\csc^2\theta - \cot^2\theta = 1$
3. $\sin\theta \tan\theta = \sec\theta - \cos\theta$
4. Simplify $\tan\theta \cot\theta - \sin^2\theta$.

Do you UNDERSTAND?

5. **Vocabulary** How does the identity $\cos^2\theta + \sin^2\theta = 1$ relate to the Pythagorean Theorem?
6. **Error Analysis** A student simplified the expression $2 - \cos^2\theta$ to $1 - \sin^2\theta$. What error did the student make? What is the correct simplified expression?