4-1 Quadratic Functions and Equations

Standards

A2.A.REI.D.6 (formerly A-REI.D.11) Explain why the x-coordinates of the points where the graphs of the equations y = f(*x*) and y = g(*x*) intersect are the solutions of the equation f(*x*) = g(*x*); find the approximate solutions using technology.

A2.F.BF.A.1 Write a function that describes a relationship between two quantities.

A2.F.BF.A.1a Determine an explicit expression, a recursive process, or steps for calculation from a context.

A2.F.BF.A.1b Combine standard function types using arithmetic operations.

Key Concepts

_____ - a function that can be written in the standard form

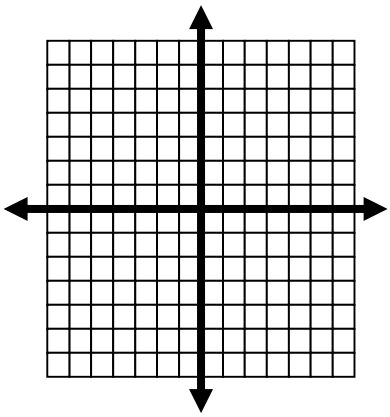
$f(x) = ax^2 + bx + c$, where $a \neq 0$

_____ - the graph of a quadratic function

_____- vertex form of a parabola where (*h*, *k*) is the vertex.

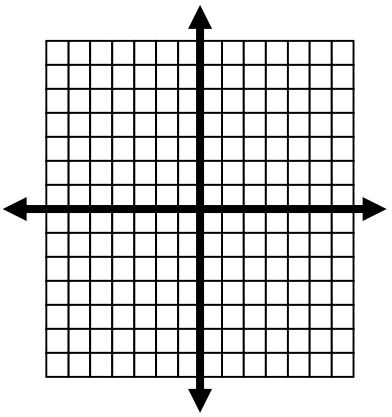
Examples

x	*f(x)*

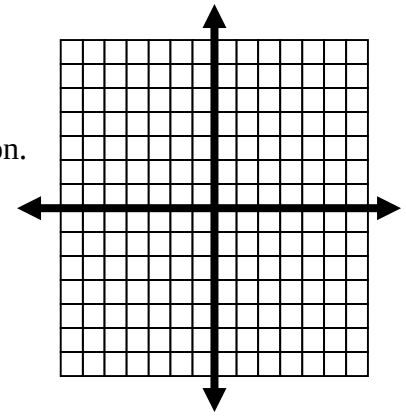


1. (I do) Graph $f(x) = x^2$
   a. Identify the vertex.
   b. Identify the axis of symmetry.
   c. Identify the maximum or minimum value.

2. (We do) Graph the translation $f(x) = x^2 - 5$
   a. Describe how the graph is a translation of the parent function $y = x^2$
   b. Identify the vertex. Is it a maximum or minimum?
   c. Identify the axis of symmetry.
   d. State the maximum or minimum value.



3. (They do) Graph the transformation $g(x) = -\frac{1}{3}x^2 + 2$

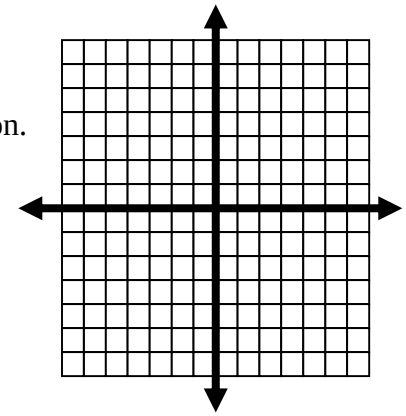


   a. Describe how the graph is a transformation of the parent function.

   b. Identify the vertex. Is it a maximum or minimum?

   c. Identify the axis of symmetry.

4. (They do) Graph the transformation $g(x) = -2(x + 1)^2 + 4$



   a. Describe how the graph is a transformation of the parent function.

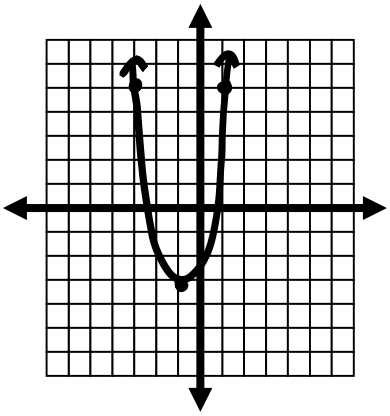
   b. Identify the vertex. Is it a maximum or minimum?

   c. Identify the axis of symmetry.

   d. What is the minimum or maximum value?

   e. State the domain and range of the function.

5. (They do) Write an equation to model the graph through vertex (-1, -3) & (-3, 5).



You do Practice 4-1: Complete your assignment on a separate sheet of paper. Show work.

1. Graph each function, describe the transformation, identify the vertex, axis of symmetry, maximum or minimum value, domain and range.

   a. $y = -x^2$ b. $y = -x^2 - 7$ c. $y = (x + 1)^2 - 4$

2. When does the graph of a quadratic function have a minimum value?

3. Describe the similarities and differences between the graphs of $y = -(x + 6)^2 - 7$ and $y = (x + 6)^2$

4. Write the equation for the parabola with vertex (-4, -4) through (-2, 0).

4-2 Standard Form of a Quadratic Function

Standards

A2.A.REI.D.6 (formerly A-REI.D.11) Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the approximate solutions using technology.

A2.F.BF.A.1 Write a function that describes a relationship between two quantities.

A2.F.BF.A.1a Determine an explicit expression, a recursive process, or steps for calculation from a context.

A2.F.BF.A.1b Combine standard function types using arithmetic operations.

Key Concepts

Standard form $y = ax^2 + bx + c$, where the axis of symmetry is $x = \frac{-b}{2a}$, the vertex is $(\frac{-b}{2a}, f(\frac{-b}{2a}))$ and the *y*-intercept is (o, *c*).

Examples

1. (I do) Graph the function $y = x^2 + 2x + 3$

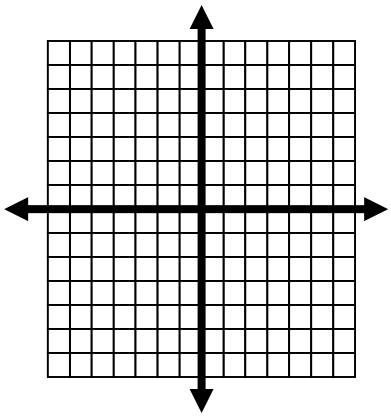
   Step 1: Identify a, b, & c.

   Step 2: Graph the axis of symmetry.

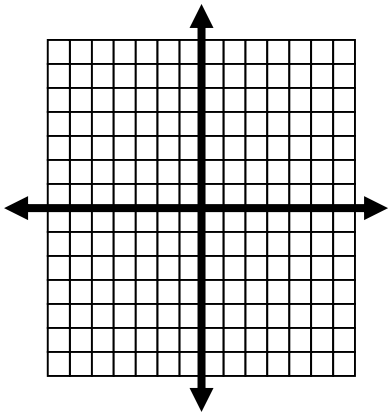
   Step 3: Find the vertex.

   Step 4: Plot the *y*-intercept and its reflection.

   Step 5: Draw the graph through the points. Label the points.



2. (We do) Graph the function $y = x^2 - 4x - 4$

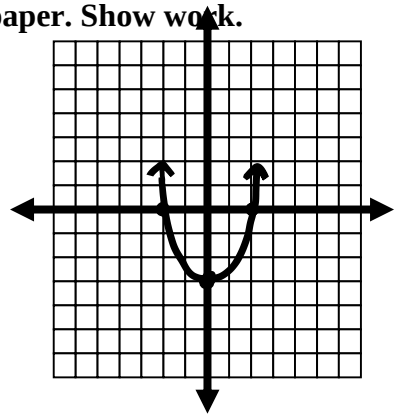


3. (I do) Convert $y = -x^2 + 4x - 5$ to vertex form.

4. (We do) Convert $y = 2(x + 3)^2 - 11$ to standard form.

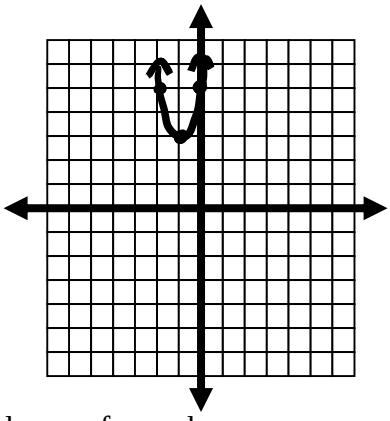
You do Practice 4-2: Complete your assignment on a separate sheet of paper. Show work.

1. Identify the vertex, axis of symmetry and maximum or minimum value for the parabola.



2. Graph $y = x^2 - 2x + 4$

3. A student graphed the function $y = 2x^2 - 4x - 3$ at the right. Find and correct the error. Then graph the function correctly.



4. Graph, state the vertex, axis of symmetry, maximum or minimum and range for each parabola.

a. $y = x^2 + 2x + 1$ **b.** $y = 3x^2 - 4x - 2$ **c.** $y = 2x^2 + 4x$

5. A model for a company's revenue from selling a software package is $R = -2.5p^2 + 500p$, where p is the price of the software. What price would maximize revenue? What is the maximum revenue?

4-3 Modeling with Quadratic Functions

Standards

A2.A.CED.A.1 (formerly A-CED.A.1) Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

A2.S.ID.B.2 Represent data on two quantitative variables on a scatter plot and describe how the variables are related. Fit a function to the data; use functions fitted to data to solve problems in the context of the data.

Examples

1. (I do) A parabola contains the points (0, 0), (-1, -2), and (1, 6). What is the equation of the parabola in standard form?

2. (We do) A player throws a basketball toward the hoop. The basketball follows a parabolic path through the points (2, 10), (4, 12), and (10, 12). Find a quadratic function to model this situation.

3. (They do) The table shows a meteorologist's predicted temperatures for this particular day.
    a. What is a quadratic model for the data?

time	temperature
8 am	52
10 am	64
12 pm	72
2 pm	78
4 pm	81
6 pm	76

    b. Use your model to predict the high temperature for the day.

You do Practice 4-3: Complete your assignment on a separate sheet of paper. Show all work.

1. Find an equation in standard form of the parabola passing through the points.
a. (1, -3), (2, 0), (3, 9) b. (3, -1), (2, -5), (4, -5)

2. A player hits a tennis ball across the court and records the height of the ball at different times, as shown in the table below.

a. Find a quadratic model for the data.
b. Use the model to estimate the height of the ball at 4 seconds.
c. What is the ball's maximum height?

Time (s)	Height (ft)
0	5.5
1	6.0
2	5.5
3	4.0

4-4 Factoring Quadratic Expressions (Part 1)

Standard

A2.A.REI.B.3 Solve quadratic equations and inequalities in one variable.
Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, knowing and applying the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± b*i* for real numbers a and b.

Key Concepts

_____ - rewriting an expression as the product of its factors. (un-distributing)

_____ – the largest quantity that is a factor of all the integers or polynomials involved.

Examples

1. Factor out the GCF.
   a. $-3x^2 - 15xy$ b. $6a^3 - 9a^2 + 12a$ c. $4x^2 - 20x + 24$

2. Factor out the GCF.
   a. $6(x + 2) - y(x + 2)$ b. $xy(y + 1) - (y + 1)$

3. Factor using grouping.
   a. $x^3 + 2x^2 - 3x - 6$ b. $x^2y + x - 4xy - 4$

4. Factor each expression (a = 1).
   a. $x^2 + 9x + 20$ b. $x^2 + 14x - 72$ c. $-x^2 + 13x - 12$

5. Factor (a ≠ 1)

a. $2a^2 + 11a + 12$

b. $4x^2 - 4x - 3$

c. $5x^2 + 28x + 32$

d. $5x^2 - 13x + 6$

You do Practice 4-4 Part 1: Complete your assignment on a separate sheet of paper. Show all work.

1. Find the GCF

a. $15x^2 - 25x$

b. $21h^3 + 35h^2 - 28h$

2. Factor

a. $x^2 + 3x + 2$

b. $x^2 + 15x + 36$

c. $-r^2 + 11r - 18$

d. $a^2 - 5a - 14$

e. $a^2 + 10a - 75$

f. $27p^2 - 9p - 18$

3. Factor

a. $3a^2 + 31a + 36$

b. $7x^2 - 8x - 12$

4-4 Factoring Quadratic Expressions (Part 2)

Standard

A2.A.REI.B.3 Solve quadratic equations and inequalities in one variable.
Solve quadratic equations by inspection (e.g., for x^2 = 49), taking square roots, completing the square, knowing and applying the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± b*i* for real numbers a and b.

Key Concepts

Perfect Square Trinomials $(a+b)^2 = a^2 + 2ab + b^2$ and $(a-b)^2 = a^2 - 2ab + b^2$

Difference of Squares $a^2 - b^2 = (a-b)(a+b)$

Examples

1. Factor
   a. (I do) $4x^2 + 12x + 9$

   b. (We do) $9x^2 + 30x + 25$

   c. (They do) $80x^2 - 200x + 125$

2. (I do) Factor using the difference of squares
   a. $x^2 - 4$ b. $4x^2 - 9$ c. $9x^2 - 36$

Practice 4-4 Part 2: Complete your assignment on a separate sheet of paper. Show all work.

1. Factor

a. $9x^2 - 1$ b. $64x^2 - 16$ c. $18h^3 - 8h$

2. Factor

a. $x^2 - 18x + 81$ b. $12x^2 + 36x + 27$ c. $4x^2 - 22x + 10$

4-5 Quadratic Equations

Standard

A2.A.REI.B.3 Solve quadratic equations and inequalities in one variable.
Solve quadratic equations by inspection (e.g., for x^2 = 49), taking square roots, completing the square, knowing and applying the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± b*i* for real numbers a and b.

Key Concepts

_____-a solution of a quadratic equation.

_____-a solution of a quadratic equation.

_____-a solution of a quadratic equation.

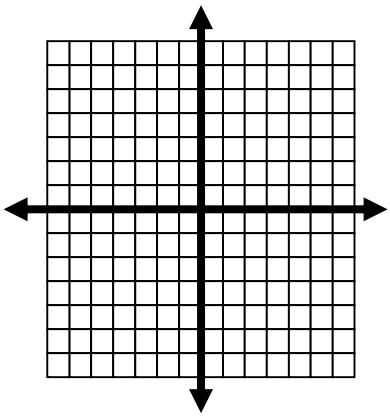
* These solutions can be found by _____, _____,

_____ or using the _____.

Examples

1. Solve the quadratic by factoring.
   a. (I do) $x^2 + x - 6 = 0$
   b. (We do) $x^2 - 7x = -12$
   c. (They do) $5x^2 - 8 = 18x$
   d. (You do) $a^2 - 4a = 0$

2. Solve the quadratic by graphing.
   a. $2x^2 + 7x = 15$



3. The function $y = -0.03x^2 + 1.60x$ models the path of a kicked soccer ball. The height is *y*, the distance is *x* and the units are meters.
   a. How high does the soccer ball go?

   b. How far does the soccer ball travel?

You do Practice 4-5: Complete your assignment on a separate sheet of paper. Show all work.

1. Solve by factoring.

a. $9x^2 - 1 = 0$ b. $x^2 + 13x = -36$ c. $h^3 - 5h^2 = 6h$

2. Solve by graphing.

a. $x^2 + 5x + 3 = 0$ b. $10x^2 + 3 = 11x$ c. $5x^2 + x = 4$

3. An object is dropped from a height of 1700 ft above the ground. The function $h = -16t^2 + 1700$ gives the object's height *h* in feet during free fall at *t* seconds. When will the object be 1000 feet above the ground?

4-6 Completing the Square

Standard

A2.A.REI.B.3 Solve quadratic equations and inequalities in one variable.
Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, knowing and applying the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± b*i* for real numbers a and b.

Key Concepts

_____- the product you obtain when you square a binomial.

Examples

1. (I do) Solve by finding the square roots
   a. $4x^2 = 36$

   b. $7x^2 - 10 = 25$

   c. $2x^2 + 9 = 13$

2. (We do) Find the value that completes the square.
   a. $x^2 - 10x$

   b. $x^2 + 6x$

3. (They do) Solve by completing the square.
   a. $x^2 + 6x + 9 = 49$

   b. $x^2 - 12x + 7 = 0$

   c. $3x^2 - 12x + 6 = 0$

4. Write the equation in vertex form y = $x^2 + 6x - 4$

You do Practice 4-6: Complete your assignment on a separate sheet of paper. Show all work.

1. Solve by taking square roots.

a. $2x^2 = 72$ b. $6x^2 = 54$

2. Solve by completing the square.

a. $x^2 + 6x - 3 = 0$ b. $x^2 + 4x + 2 = 0$

4-7 The Quadratic Formula

Standard

A2.A.REI.B.3 Solve quadratic equations and inequalities in one variable.
Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, knowing and applying the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± b*i* for real numbers a and b.

Key Concepts

_____-the value of the expression $b^2 - 4ac$

_____, expressed as $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$ can be used to solve quadratic functions.

- If $b^2 - 4ac > 0$, there are 2 real solutions.
- If $b^2 - 4ac = 0$, there is 1 real solution.
- If $b^2 - 4ac < 0$, there are 0 real solutions.

Examples

1. (I do) What are the solutions to $3x^2 + 23x = -40$? Use the Quadratic Formula.

2. (We do) What are the solutions to $3x^2 + 2x - 4 = 0$? Use the Quadratic Formula.

3. (They do) What is the number of real solutions of $-2x^2 - 3x + 7 = 0$? Use the discriminant.

4. (They do) CHS is selling pumpkin seeds as a fundraiser for the Halloween dance. The total profit *p* depends on the amount *x* that CHS charges for each box of seeds. The equation $p = -0.5x^2 + 25x - 150$ models the profit. What is the smallest amount you can charge and make a profit of at least $150?

You do Practice 4-7: Complete your assignment on a separate sheet of paper. Show all work.

1. Solve using the Quadratic Formula

a. $x^2 - 5x - 7 = 0$ **b.** $2x^2 - 5x - 3 = 0$

2. Without solving, find the discriminant and determine the number of real solutions.

a. $-2x^2 + 3x + 7 = 0$ **b**. $x^2 - 6x + 9 = 0$

3. The bakery sells more cupcakes when the prices are lower, but then its profit changes. The function $y = -1000x^2 + 1100x - 2.5$ models the bakery's daily profit, *y* in dollars, from selling cupcakes, where *x* is the price of the cupcakes in dollars. What's the highest price the bakery can charge and make a profit of at least $200?

4-8 Complex Numbers

Standard

A2.A.REI.B.3 Solve quadratic equations and inequalities in one variable.
Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, knowing and applying the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± b*i* for real numbers a and b.

Key Concepts

_____ - the imaginary unit defined as the number whose square is -1. $\boldsymbol{i} = \sqrt{-1}$

_____ = -1

_____- a number defined as *a* + *bi*, where *a* is real and *bi* is imaginary.

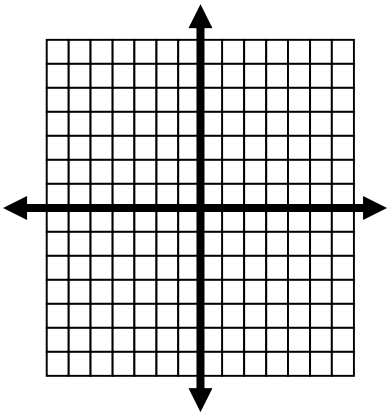
_____-number pairs of the form *a* + *bi* and *a* - *bi*

Examples

1. (I do) Simplify
   a. $\sqrt{-16}$ b. $\sqrt{-11}$ c. $\sqrt{-20}$ d. $\sqrt{-125}$

2. (We do) Graph each complex number and find its absolute value.
   a. $-5 + 3i$

   b. $2i$



3. (You do) Simplify using addition or subtraction.
   a. $(4 - 3i) + (-4 + 3i)$ b. $(5 - 3i) - (-2 + 4i)$ c. $(7 - 2i) + (3 + i)$

4. (They do) Simplify using multiplication.
   a. $(6i)(1 - 5i)$ b. $(7i)(3i)$ c. $(4 + 3i)(-1 - 2i)$

d. $(2-4i)^2$ e. $(-4+5i)(-4-5i)$

5. (I do) Divide.
   a. $\frac{9+12i}{3i}$ b. $\frac{2+3i}{1-4i}$

6. (We do) Solve.
   a. $9x^2+54=0$ b. $x^2+5=4x$ c. $3x^2-x+2=0$

You do Practice 4-8: Complete your assignment on a separate sheet of paper. Show work.

1. Simplify

a. $\sqrt{-75}$ **b**. $\sqrt{-216}$ **c**. $(4-2i)-(3+i)$

d. $(2+i)(4-5i)$ **e**. $\frac{4-i}{6i}$ **f.** $(9+4i)^2$

2. Find the absolute value of $4-3i$.

3. Solve

a. $x^2+16=0$ **b**. $2x^2-4x=-7$

4. Error Analysis. Describe and correct the error made in simplifying $(4+7i)(4-7i)$.

$$\begin{aligned}(4+7i)(4-7i) &= 16+28i-28i+49i^2\\ &= 16-49\\ &= -33\end{aligned}$$

4-9 Quadratic Inequalities & Quadratic Systems

Standard

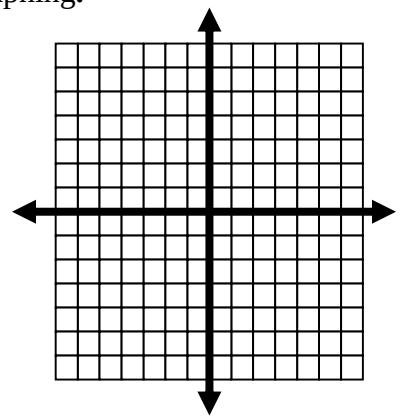
A2.A.REI.B.3 Solve quadratic equations and inequalities in one variable.
Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, knowing and applying the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± b*i* for real numbers a and b.

Examples

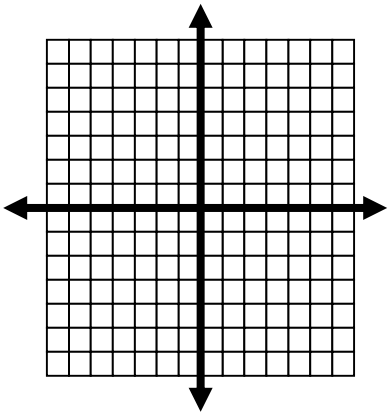
1. (I do) Solve each inequality algebraically. Graph your solution on a number line.
   a. $2x^2 - 14x < 0$ b. $2x^2 - 14x > 0$

2. (We do) Solve $\frac{1}{4}(x-2)^2 - 1 > 0$ and $\frac{1}{4}(x-2)^2 - 1 < 0$ by graphing**.**

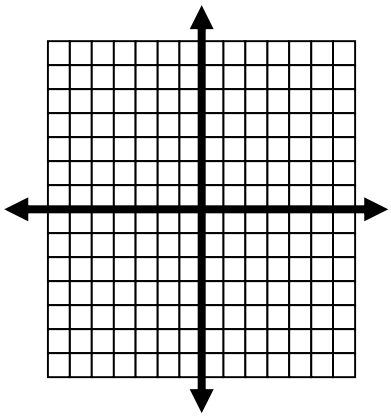
3. (We do) Solve the system by substitution. $\begin{cases} y = x + 6 \\ y = -x^2 + 5x + 6 \end{cases}$



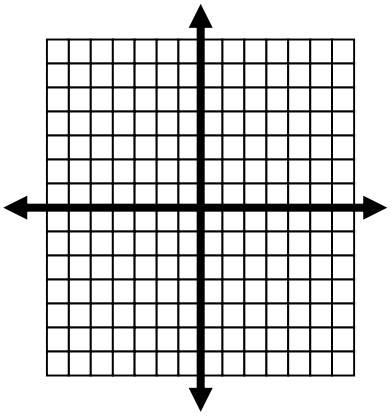
4. (They do) Solve the system by graphing. $\begin{cases} y = x - 3 \\ y = x^2 - 2x + 1 \end{cases}$



5. (They do) Solve the system by graphing. $\begin{cases} y = -x^2 - x + 12 \\ y = x^2 + 7x + 12 \end{cases}$



6. (They do) Solve the system of by graphing. $\begin{cases} y < -x^2 - 9x - 2 \\ y > x^2 - 2 \end{cases}$



Practice 4-9: Complete your assignment on a separate sheet of paper. Show all work.

1. Solve the inequality. Graph your solution on the number line.
   a. $x^2 < 36$ b. $x^2 - 3x - 18 > 0$

2. Solve the system by substitution. $\begin{cases} y = x + 1 \\ y = x^2 - 2x + 3 \end{cases}$

3. Solve the system by substitution. $\begin{cases} y = x - 2 \\ y = 2x^2 - 5x + 2 \end{cases}$

4. Solve the system by graphing. $\begin{cases} y > 2x^2 + x + 3 \\ y < -x^2 - 4x + 1 \end{cases}$

5. Reasoning. How many points of intersection can the graphs of the following types of functions have? Draw graphs to justify your answers.
   a. a linear function and a quadratic function
   b. two quadratic functions
   c. a linear function and an absolute value function