5-1 Polynomial Functions

Standards

A2.A.APR.A.2 (formerly A-APR.A.3) Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.
A2.F.IF.B.5 (formerly F.IF.C.9) Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Key Concepts

_________________ – a real number, a variable, or a product of a real number and one or more variables with whole number exponents

________________________– a monomial or the sum of monomials

_____________________________ – the largest exponent of any term of the polynomial

____– arranges the terms by degree in a descending numerical order $P(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0$ where n is a nonnegative integer and $a_n, \ldots, a_0$ are real numbers.

_____________________– direction of the graph to the far left and to the far right.

Degree	Name
0	constant
1	linear
2	quadratic
3	cubic
4	quartic
5	quintic

of terms	Name
1	monomial
2	binomial
3	trinomial
4 or more	polynomial

End Behavior

	Even Degree	Odd Degree
Leading coefficient positive	up and up	down and up
Leading coefficient negative	down and down	up and down

Examples

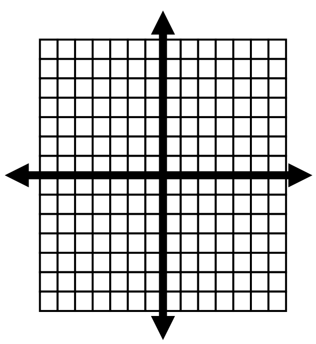
1. (I do, We do) Write each polynomial in standard form. Then classify it according to its' degree and the number of terms.

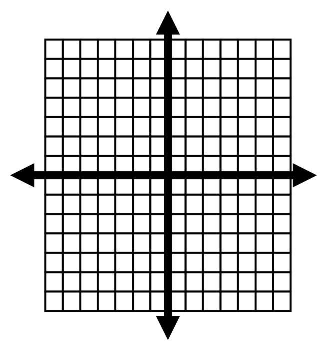
   a. $3x^4 + x^2 - 1$ b. $-x^2 + 2x^3 - x + 1$ c. $(2x^3 + x^5) - (6x^5 + 2x^3)$

   c. $(x^2 - 3x + 4)(-5x^2 + 8x + 3)$ d. $(x^2 + 4)(x^2 - 4)$

2. (They do) Describe the shape, turning points, end behavior, domain and range on the following graphs. Check using the calculator.

   a. $y = 4x^3 - 3x$ b. $y = -2x^4 + 8x^3 - 8x^2 + 2$





You do Practice 5-1: Complete your assignment on a separate sheet of paper. Show work.

1. Write the polynomial in standard form. Classify each polynomial by degree and by number of terms.

   a. $5x^3$ b. $4x + 6x^2 + x - 2$ c. $7x - 3 + 2x^2$

   d. $x^2 - 2x + 4x^3 - 1$ e. $(x + 2)^2$ f. $(x^3 - 2x + 1)(x - 1)$

2. Describe the shape, turning points, end behavior domain and range of the graph for each function.

   a. $-7x^3 + 8x^2 + x$ b. $1 - 4x - 6x^3 - 15x^6$

3. **Error Analysis.** Your friend claims the graph of the function $y = 4x^3 + 4$ has only one turning point and the end behavior is up and down. Is your friend right? If not describe the error your friend made and give the correct number of turning points end behavior.

5-2 Polynomials, Linear Factors, and Zeros (Part 1)

Standards

A2.A.APR.A.2 (formerly A-APR.A.3) Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.
A2.F.IF.A.2 (formerly F-IF.B.6) Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
A2.F.IF.B.5 (formerly F-IF.C.9) Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Key Concepts

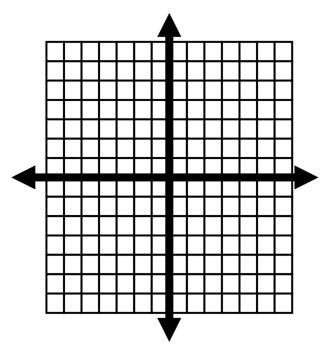
The following are equivalent statements about a real number *b* and a polynomial P(*x*)

- *x* – *b* is a ______________________of the polynomial P(*x*)
- *b* is a _________________of the polynomial function y = f(*x*)
- *b* is a ______________________________of the polynomial equation f(*x*) = 0
- *b* is an __________________________of the graph y = f(*x*)

Examples

1. (I do) Write $3x^3 - 18x^2 + 24x$ in factored form.

2. (We do) Find the zeros for $y = (x-1)(x+1)(x+3)$. Then graph the function. State the domain and range.



3. (They do) Factor. Then find the zeros.
   a. $y = x^3 - 2x^2 - 15x$
   b. $y = 2x^3 - 5x^2 - 3x$

4. (They do) Write a polynomial with the given zeros.
   a. -2, -2, 2

   b. -3, 1, 2, 3

5. Given the function $f(x) = x^2 - 5x - 1,$ determine the average rate of change of the function over the interval $-2 \leq x \leq 5$

You do Practice 5-2 Part 1: Complete your assignment on a separate sheet of paper. Show all work.

1. Find the zeros for each function.
   a. $y = x(x - 6)$ b. $y = (2x + 3)(x - 1)$ c. $y = x^3 - 4x^2 - 21x$

2. Write a polynomial function in standard form with the zeros $x =$ -2, 1, -1.

3. Error Analysis. Your friend says a function with zeros 3 and -1 is $f(x) = x^2 + 2x - 3$. Is your friend correct? If not find and correct the error.

5-2 Polynomials, Linear Factors, and Zeros (Part 2)

Standards

A2.A.APR.A.2 (formerly A-APR.A.3) Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.

A2.F.IF.A.2 (formerly F-IF.B.6) Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

A2.F.IF.B.5 (formerly F-IF.C.9) Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Key Concepts

______________________- the greatest *y*-value of the points in a region of graph

___________________________- the least *y*-value among nearby points on a graph

A multiple zero has a ____________________________________ equal to the number of times a zero occurs.

Examples

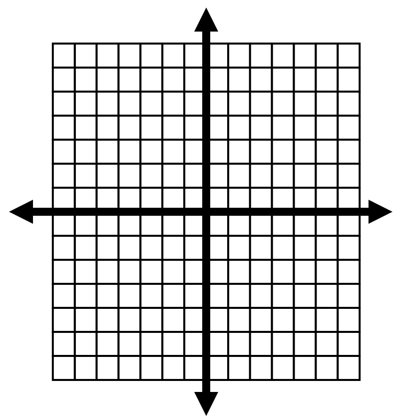
1. (You do) Find the multiplicity of the zeros for $y = x^4 - 2x^3 - 8x^2$.

2. (We do) Identify the relative maximum and minimum for $f(x) = x^3 + 3x^2 - 24x$

3. (They do) What are the relative maximum and minimum for $f(x) = -4x^3 + 12x^2 + 4x - 12$?

4. (They do) The volume in cubic feet of a DVD holder can be expressed as $V(x) = -x^3 - x^2 + 6x$. The length is expressed as $x - 2$. Assume the height is greater than the width.
   a. Factor and find the linear expressions for the height and width.

   b. Graph the function. Find the *x*-intercepts. What do the *x*-intercepts represent? Use an appropriate solution to find the actual dimensions of the DVD holder.



   c. What is a realistic domain and range for the function?

   d. What is the maximum volume of the DVD holder?

You do Practice 5-2 Part 2: Complete your assignment on a separate sheet. Show all work.

1. Find the zeros of each function. State the multiplicity of multiple zeros.
   a. $y = x(x-3)^2$
   b. $y = 3x^4 + 24x^3 + 48x^2$

2. Find the relative maximum and relative minimum of the graph of each function.
   a. $f(x) = x^3 + 4x^2 - 5x$
   b. $f(x) = -x^3 - 7x^2 + 7x + 15$

3. A rectangular box is $2x + 3$ units long and $2x - 3$ units wide, and $3x$ units high. What is the volume expressed as a polynomial in standard form?

5-3 Solving Polynomial Equations

Standards

A2.A.APR.A.2 (formerly A-APR.A.3) Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
A2.A.APR.B.3 Know and use polynomial identities to describe numerical relationships.
A2.F.IF.A.1 (formerly F-IF.B.4) For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
A2. F.BF.A.1 (formerly F-BF.A.1) Write a function that describes a relationship between two quantities.
A2.F.BF.B.3 (Formerly F-BF.B.3) Identify the effect on the graph of replacing f(*x*) by f(*x*) + *k*, *k* f(*x*), f(*kx*), and f(*x* + *k*) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
A2.A.REI.D.6 (formerly A-REI. D.11)_ Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the approximate solutions using technology.
A2.N.Q.A.1 (formerly N-Q.B.2) Identify, interpret, and justify appropriate quantities for the purpose of descriptive modeling. Descriptive modeling refers to understanding and interpreting graphs; identifying extraneous information; choosing appropriate units; etc.

Key Concepts

Factoring by Grouping- $ax + ay + bx + by = a(x + y) + b(x + y) = (a + b)(x + y)$

Sum of Cubes- $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Difference of Cubes- $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Examples

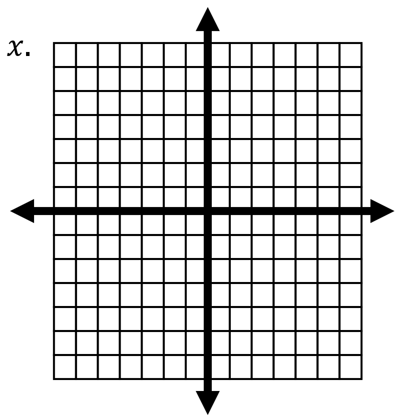
1. What are the real or imaginary solutions of each polynomial equation?
   a. (I do) $2x^3 - 5x^2 = 3x$
   b. $3x^4 + 12x^2 = 6x^3$
   c. (We do) $x^4 + 3x^2 - 4 = 0$
   d. $x^3 - 27 = 0$

e. (They do) $x^3 + 3x^2 + 4x + 12 = 0$

f. $x^3 + 2x^2 - 3x - 6 = 0$

2. (They do) Find the real roots by graphing $x^3 + 5 = 4x^2 + x$.

   a. State the domain and range.



3. (They do) Three close friends were all born on July 4th. Stacy is one year younger than Nikki. Nikki is 2 years younger than Amir. This year, the product of their ages was 2300 more than the sum of their ages. How old is each friend?

You do Practice 5-3: Complete your assignment on a separate sheet of paper. Show all work.

1. Solve each equation by factoring.

a. $x^3 - 64 = 0$

b. $2x^3 + 8x^2 + 4x = -16$

c. $2x^3 + 2x^2 - 4x = 0$

d. $x^4 - 2x^2 = 8$

2. Find the real solutions of $4x^3 = 4x^2 + 3x$ by graphing.

3. The Johnson twins were born 2 years after their older sister. This year, the product of the three sibling's ages is exactly 4558 more than the sum of their ages. How old are the twins?

5-4 Dividing Polynomials (Part 1)

Standards

A2.A.APR.A.1 (formerly A-APR.A.2) Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by $x - a$ is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).
A2.A.APR.C.4 (formerly A-APR.C.6) Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.

Key Concepts

The Divisor Algorithm**-**You can divide polynomial P(x) by polynomial D(x) to get the quotient Q(x) and a remainder R(x). If R(x) = 0, then D(x) and Q(x) are factors of P(x).

Examples

1. (I do) Divide $x^2 + 2x - 30$ by $x - 5$

2. (We do) Divide $(4x^2 + 23x - 16) \div (x + 5)$

3. (They do) Divide $(x^3 - 7x^2 - 36) \div (x - 2)$

4. (They do) Determine whether $x + 2$ is a factor of the polynomial $x^2 + 10x + 16$

You do Practice 5-4 Part 1: Complete your assignment on a separate sheet of paper. Show all work.

1. Divide using long division.
   a. $(2x^2 + 7x + 11) \div (x + 2)$
   b. $(x^3 + 5x^2 + 11x + 15) \div (x + 3)$
   c. $(9x^3 - 15x^2 + 4x) \div (x - 3)$

2. Determine whether $x + 1$ is a binomial factor of $x^3 + 4x^2 + x - 6$.

5-4 Dividing Polynomials (Part 2)

Standards

A2.A.APR.A.1 (formerly A-APR.A.2) Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by $x - a$ is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).
A2.A.APR.C.4 (formerly A-APR.C.6) Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.

Key Concepts

____– simplifies long division for dividing by a linear expression $x - a$.

________________________________ states if you divide a polynomial P(x) by $x - a$, then the remainder is P(a).

Examples

1. (I do) Use synthetic division to divide
   a. $(5x^3 - 6x^2 + 4x - 1) \div (x - 3)$
   b. $(x^3 - 57x + 56) \div (x - 7)$

2. (We do) Given that $P(x) = x^5 - 2x^3 - x^2 + 2$, use the remainder theorem to find $P(3)$.

3. (They do) Use synthetic division to completely factor $x^3 + 2x^2 - 5x - 6$, given $(x + 1)$ is a factor.

4. (They do) When a polynomial is divided by $(x - 5)$, the quotient is $5x^2 + 3x + 12$ with remainder of 7. Find the polynomial.

You do Practice 5-4 Part 2: Complete your assignment on a separate sheet of paper. Show all work.

1. Divide using synthetic division.
   a. $(x^3 + 3x^2 - x - 3) \div (x - 1)$
   b. $(x^3 - 7x^2 - 7x + 20) \div (x + 4)$
   c. $(x^3 + 27) \div (x + 3)$

2. Use synthetic division to completely factor $y = 2x^3 - 7x^2 - 5x + 4$ given that $(x + 1)$ is a factor.

3. When a polynomial is divided by $(x + 3)$, the quotient is $x^2 - 7x + 10$ with remainder of -26. Find the polynomial.

5-5 Theorems about Roots of Polynomial Equations (Part 1)

Standards

A2.A.APR.A.1 (formerly A-APR.A.2) Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by *x* – *a* is p(*a*), so p(*a*) = 0 if and only if (*x* – a) is a factor of p(*x*).
A2.A.APR.C.4 (formerly A-APR.C.6) Rewrite simple rational expressions in different forms; write a(*x*)/b(*x*) in the form q(*x*) + r(*x*)/b(*x*), where a(*x*), b(*x*), q(*x*), and r(*x*) are polynomials with the degree of r(*x*) less than the degree of b(*x*), using inspection, long division, or, for the more complicated examples, a computer algebra system.

Key Concepts

Rational Root Theorem- Let $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_o$. Integer roots must be factors of a_o. The rational roots are a reduced form of $\frac{p}{q}$, where p is an integer factor of a_o and q is an integer factor of a_n.

Examples

1. (I do) What are the possible rational roots of $2x^3 - x^2 + 2x + 5 = 0$?

2. (We do) List all possible roots using the Rational Root Theorem. Then find the rational roots of $3x^3 + 7x^2 + 6x - 8 = 0$?

3. (They do) List all possible roots using the Rational Root Theorem. Then find the rational roots of $15x^3 - 32x^2 + 3x + 2 = 0$?

You do Practice 5-5 Part 1: Complete your assignment on a separate sheet of paper. Show all work.

1. Use the rational root theorem to list all the possible roots. Then find the rational roots.

a. $x^2 + x - 2 = 0$

b. $4x^3 + 12x^2 + x + 3 = 0$

c. $3x^4 + 2x^2 - 12 = 0$

5-5 Theorems about Roots of Polynomial Equations (Part 2)

Standards

A2.A.APR.A.1 (formerly A-APR.A.2) Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by $x - a$ is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).
A2.A.APR.C.4 (formerly A-APR.C.6) Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.

Key Concepts

- Conjugate Root Theorem
  If P(x) is a polynomial with rational coefficients, then the irrational roots of P(x) occur in pairs. If a - √b is an irrational root, then a + √b is also a root.
- Imaginary Root Theorem
  If the imaginary number a + bi is a root of a polynomial equation with real coefficients, then the conjugate a – bi also is a root.
- Descartes' Rule of signs: The number of positive roots is either equal to the number of sign changes of P (x) or less by an even number. The number of negative roots is either equal to the number of sign changes of P (-x) or less by an even number.

Examples

1. (I do) P(x) is a polynomial with rational coefficients. If $\sqrt{2}$ and 1 + i are roots of the polynomial equation. What are the other roots?

2. (We do) What is the third-degree polynomial function with rational coefficients that has -4 and 2i as roots?

3. (They do) What is a polynomial function with rational coefficients that has $1-\sqrt{5}$ and $2 + i$ as roots.

4. (They do) What does Descartes' Rule of signs tell you about the number of positive and negative real roots for each function.
   a. $x^3 - x^2 + 1$

   b. $4x^5 - x^4 - x^3 + 6x^2 - 5$

You do Practice 5-5 Part 2: Complete your assignment on a separate sheet of paper. Show all work.

1. Write a polynomial function with rational coefficients so that $P(x)$ has the given roots.
   a. -10*i*

   b. 2 and 3*i*

   c. 1-2*i* and $\sqrt{10}$

2. What does Descartes' Rule of signs say about the number of positive real roots for each function?
   a. $P(x) = x^2 + 5x + 6$
   b. $P(x) = 8x^3 + 2x^2 - 14x + 5$

5-6 The Fundamental Theorem of Algebra

Standards

A2.A.APR.A.1 (formerly A-APR.A.2) Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by $x – a$ is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).
A2.A.APR.C.4 (formerly A-APR.C.6) Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.

Key Concepts

The Fundamental Theorem of Algebra- If P(x) is a polynomial of degree $n \geq 1$, the P(x) = 0 has exactly n roots, including multiple and complex roots.

Examples

1. (I do) What are all the roots of $x^5 - x^4 - 3x^3 + 3x^2 - 4x + 4 = 0$

2. (We do) What are all the zeros of $f(x) = x^4 + x^3 - 7x^2 - 9x - 18$?

3. (They do) What are all the roots of $2x^4 - 3x^3 - x - 6 = 0$

You do Practice 5-6: Complete your assignment on a separate sheet of paper. Show all work.

1. State the number of roots for each equation. Then find all roots.

a. $5x^3 - 24x^2 + 41x - 20 = 0$

b. $x^4 + x^3 - 7x^2 - 9x - 18 = 0$

c. $x^4 - 2x^3 + x^2 - 2x = 0$

5-8 Polynomial Models in the Real World

Standard

A2.F.IF.B.3 (formerly F-IF.C.7c) Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology. b. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

Key Concepts

________________________________-for any *n* + 1 points in the coordinate plane that pass the vertical line test, there is a unique polynomial of degree at most n that fits the points perfectly.

*The closer r^2 gets to 1, the better the fit.

Examples

1. (I do) What polynomial function has a graph that passes through the four points (0, -3), (1, -1), (2, 5) and (-1, -7)?

2. (We do) Find a polynomial whose graph passes through (-2, -16), (3, 11) and (0, 2).

3. (They do) For the data below that examines U.S. Federal Spending, compare two models and determine which one best fits the data. Which model seems more likely to represent the data set over time?

Year	**Total (billions)**
1965	630
1980	1300
1995	1950
2005	2650

You do Practice 5-8: Complete your assignment on a separate sheet of paper. Show all work.

1. Find a polynomial function whose graph passes through each set of points.

a. (-1, 8), (5, -4) and (7, 8)

b. (-1, -15), (1, -7) and (6, -22)

c. (-1, 9), (0, 6), (1, 5) and (2, 18)

2. For the data below that examines World Population, compare two models and determine which one best fits the data. Which model seems more likely to represent the data set over time?

Year	**Price (thousands**
1991	149
1995	158
2000	207

5-9 Transforming Polynomial Functions

Standard

A2.F.IF.B.3 (formerly F-IF.C.7c) Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology. b. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

Key Concepts

______________________________- a function of the form $y = a(x^b)$ where a and b are non zero real numbers

____- the constant a in a power function

______________________________- general form of a cubic function

Examples

1. (I do) What is an equation of the graph of $y = x^3$ under a vertical compression by a factor of $\frac{1}{2}$ followed by a reflection across the *x*-axis, a horizontal translation 3 units to right and a vertical translation 4 units up?

2. (We do) Find all the real zeros of the function $y = -27(x-2)^3 + 8$

3. (They do) What is a quartic function in standard form with only two real zeros, $x = 5$ and $x = 9$

You do Practice 5-9: Complete your assignment on a separate sheet of paper. Show all work.

1. Find all the real zeros of each function.
   a. $y = -(x+3)^3 + 1$

2. Determine the equation of a cubic function obtained from the parent function $y = x^3$ after the following transformations.
   a. Vertical stretch by a factor of 3, reflection across the *x*-axis, vertical translation 2 units up and horizontal translation 1 unit right.

   b. Vertical compression by a factor of $\frac{1}{3}$, vertical translation of 4 units down and a horizontal translation of 2 units left.

3. Find a quartic function in standard form with the given *x*-values as its only real zeros.
   a. $x = 2$ and $x = -1$