

6-0 Properties of Exponents

Standards

A2.N.RN.A.1 (formerly N-RN.A.1) Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

A2.N.RN.A.2 (formerly N-RN.A.2) Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Key Concepts

Properties of Exponents- Assume that no denominator is equal to zero and m and n are integers.

- $a^0 = 1$
- $a^{-n} = \frac{1}{a^n}, a \neq 0$
- $a^m \cdot a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$
- $(ab)^n = a^n b^n$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$
- $(a^m)^n = a^{mn}$

Examples

- Simplify each expression using only positive exponents.

a. $(3a^4)(-2a^{-5})$ b. $(-3x^{-3}y^4)^2$ c. $\frac{(x^2y)^0}{2x^{-3}}$

d. $\frac{6a^3b^{-2}c^5}{ab^{-3}c^3}$ e. $\left(\frac{2x^2y^{-2}}{3}\right)^3$ f. $\left(\frac{3r^{-2}s^3t^0}{3rs}\right)^{-3}$

You do Practice 6-0: Complete your assignment on a separate sheet of paper. Show all work.

- Simplify. Your exponents should only include positive exponents.

a. $(x^{-2}y^{-3})^4$ b. $(x^4)^{-3}(2x^4)$ c. $\frac{2y^3 \cdot 3xy^3}{3x^2y^4}$

d. $\frac{x^3y^3z^2}{3x^2y^4}$ e. $\frac{3x^2y^2}{2x^{-1}(4xy^2)}$ f. $\frac{2x^2y^4 \cdot 4x^2y^4 \cdot 3x}{3x^{-3}y^2}$

6-1 Roots and Radical Expressions

Standards

A2.N.RN.A.1 (formerly N-RN.A.1) Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

A2.N.RN.A.2 (formerly N-RN.A.2) Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Key Concepts

_____ - For any real numbers a and b , and any positive integer n , if $a^n = b$, then a is an n th root of b .

- If n is _____, there is _____ real n th root.
- If n is _____, there are _____ real n th roots.

_____ - the number under the radical.

_____ - the degree of the root.

_____ - the positive root when the number has two real roots.

_____ - $\sqrt[n]{a^n} = \begin{cases} a & \text{if } n \text{ is odd} \\ |a| & \text{if } n \text{ is even} \end{cases}$

Examples

1. (I do) Find all real cube roots.

a. 0.027

b. -125

c. $\frac{1}{64}$

2. (We do) Find all real fourth roots.

a. 625

b. -0.0016

c. $\frac{81}{625}$

3. (We do) What is each principal real number root?

a. $\sqrt[3]{-27}$

b. $\sqrt{0.09}$

c. $\sqrt[4]{-16}$

d. $\sqrt{(-3)^2}$

4. (We do) Simplify each radical expression. Use absolute value symbols as needed.

a. $\sqrt{16x^8}$

b. $\sqrt[3]{27a^3b^3}$

c. $\sqrt[4]{x^{16}y^4}$

d. $\sqrt[4]{81(x+y)^8}$

5. (They do) Find all real solutions.

a. $x^2 = 81$

b. $x^3 = 27$

c. $x^4 = \frac{256}{625}$

d. $x^4 = -16$

6. (They do) The voltage V of an audio system's speaker can be represented by $V = 4\sqrt{P}$, P is the power of the speaker.

a. An engineer wants to design a speaker with 400 watts of power. What would the voltage be?

b. Casey wants to buy an audio system's speaker with a voltage of 100. What would be the power of the speaker in watts?

(You do) Practice 6-1: Complete your assignment on a separate sheet of paper. Show work.

1. Find all the real square roots.

a. 625

b. $\frac{16}{81}$

2. Find all the real cube roots.

a. -216

b. 0.027

3. Find all the real fourth roots.

a. -1296

b. 0.2401

4. Find each principal real number root.

a. $\sqrt{400}$

b. $-\sqrt[4]{256}$

c. $\sqrt[3]{-729}$

5. Simplify each radical expression. Use absolute value symbols when needed.

a. $\sqrt{25x^6}$

b. $\sqrt[3]{343x^9y^{12}}$

c. $\sqrt[4]{16x^{16}y^{20}}$

6. **Reasoning.** Explain how you know whether or not to include the absolute value symbol on your root.

6-2 Multiplying and Dividing Radical Expressions

Standards

A2.N.RN.A.1 (formerly N-RN.A.1) Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

A2.N.RN.A.2 (formerly N-RN.A.2) Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Key Concepts

Multiplying Radicals $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$

Dividing Radicals $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

_____ – where the radicand of $\sqrt[n]{a}$ has a perfect nth power among its factors that can be reduced to a simpler form as much as possible.

Examples

1. (I do) Can you simplify the product? If so, simplify.

a. $\sqrt[3]{6} \cdot \sqrt{2}$

b. $\sqrt{18} \cdot \sqrt{2}$

c. $\sqrt[3]{4} \cdot \sqrt[3]{-2}$

2. (I do) Simplify.

a. $\sqrt[3]{54x^7}$

b. $\sqrt{128x^7}$

c. $\sqrt[3]{135x^8y^5}$

3. (We do) Simplify.

a. $\sqrt{72x^3y^2} \cdot \sqrt{10x^2y^3}$

b. $2\sqrt[3]{25xy^8} \cdot 3\sqrt[3]{5x^4y^3}$

4. (They do) Divide and simplify.

a. $\frac{\sqrt[3]{-81}}{\sqrt[3]{3}}$

b. $\frac{\sqrt{18x^5}}{\sqrt{2x^3}}$

c. $\frac{\sqrt[3]{162y^4}}{\sqrt[3]{3y^2}}$

5. (They do) Rationalize the denominator. Simplify

a. $\sqrt{\frac{5}{3x}}$

b. $\frac{\sqrt{x^3}}{\sqrt{3x^2y}}$

c. $\sqrt[3]{\frac{5}{4y}}$

You do: Practice 6-2: Complete your assignment on a separate sheet of paper. Show all work.

1. Multiply, if possible.

a. $\sqrt[3]{4} \cdot \sqrt[3]{6}$

b. $\sqrt{5} \cdot \sqrt{8}$

c. $\sqrt[3]{6} \cdot \sqrt[4]{9}$

2. Simplify. Assume all variables are positive. Use absolute value symbols when needed.

a. $\sqrt[3]{27x^6}$

b. $\sqrt{48x^3y^4}$

c. $\sqrt[5]{128x^2y^{25}}$

3. Multiply and simplify. Assume all variables are positive. Use absolute value symbols when needed.

a. $\sqrt{12} \cdot \sqrt{3}$

b. $\sqrt[4]{7x^6} \cdot \sqrt[4]{32x^2}$

c. $2\sqrt[3]{6x^4y} \cdot 3\sqrt[3]{9x^5y^2}$

4. Divide and simplify. Assume all variables are positive. Use absolute value symbols when needed.

a. $\frac{\sqrt[4]{405x^8y^2}}{\sqrt[4]{5x^3y^2}}$

b. $\frac{\sqrt[3]{75x^7y^2}}{\sqrt[4]{25x^4}}$

5. **Error Analysis.** Your classmate simplified $\sqrt{5x^3} \cdot \sqrt[3]{5xy^2}$ to $5x^2y$. What mistake did she make? What is the correct answer?

6-3 Binomial Radical Expressions

Standards

A2.N.RN.A.1 (formerly N-RN.A.1) Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

A2.N.RN.A.2 (formerly N-RN.A.2) Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Key Concepts

_____ - radical expressions that have the same index and the same radicand.

_____ - multiply the numerator and denominator of the fraction by the conjugate of the denominator.

Examples

1. (I do) Simplify each expression.

a. $3\sqrt{5x} - 2\sqrt{5x}$

b. $12^3\sqrt{7xy} - 8^5\sqrt{7xy}$

c. $3x^3\sqrt{2xy} + 4x^3\sqrt{2xy}$

2. (I do) Simplify

a. $\sqrt{12} + \sqrt{75} - \sqrt{3}$

b. $\sqrt[3]{250} + \sqrt[3]{34} - \sqrt[3]{16}$

3. (We do) Simplify

a. $(4 + 2\sqrt{2})(5 + 4\sqrt{2})$

b. $(3 - \sqrt{7})(5 + \sqrt{7})$

c. $(6 + \sqrt{12})(6 - \sqrt{12})$

4. (They do) Rationalize the denominator

a. $\frac{3\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

b. $\frac{1-\sqrt{8}}{2-\sqrt{8}}$

You do: Practice 6-3: Complete your assignment on a separate sheet of paper. Show all work.

1. Simplify.

a. $\sqrt{18} + \sqrt{32}$

b. $\sqrt[4]{324} - \sqrt[4]{2500}$

c. $\sqrt[3]{192} + \sqrt[3]{24}$

2. Multiply.

a. $(3-\sqrt{6})(2-\sqrt{6})$

b. $(5+\sqrt{5})(1-\sqrt{5})$

c. $(7-\sqrt{2})(7+\sqrt{2})$

3. Rationalize each denominator. Simplify the answer.

a. $\frac{3}{2+\sqrt{6}}$

b. $\frac{7+\sqrt{5}}{6-\sqrt{5}}$

4. **Error Analysis.** A classmate simplified the expression $\frac{1}{1-\sqrt{2}}$ using the steps shown. What mistake did your classmate make? What is the correct answer?

$$\begin{aligned} & \frac{1}{1-\sqrt{2}} \cdot \frac{1-\sqrt{2}}{1-\sqrt{2}} \\ &= \frac{1-\sqrt{2}}{1-2} = \frac{1-\sqrt{2}}{-1} = -1+\sqrt{2} \end{aligned}$$

6-4 Rational Exponents

Standards

A2.N.RN.A.1 (formerly N-RN.A.1) Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

A2.N.RN.A.2 (formerly N-RN.A.2) Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Key Concepts

Rational Exponent $a^{\frac{1}{n}} = \sqrt[n]{a}$ and $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

All the properties of **integer** exponents also apply to **rational** exponents.

Examples

1. (I do) Simplify

a. $64^{\frac{1}{2}}$

b. $216^{\frac{1}{3}}$

c. $7^{\frac{1}{2}} \cdot 7^{\frac{1}{2}}$

2. (I do) Convert the expressions to radical form.

a. $x^{\frac{3}{7}}$

b. $y^{3.5}$

c. $w^{-\frac{3}{4}}$

d. $a^{-0.2}$

3. (We do) Convert the expressions to exponential form.

a. \sqrt{y}

b. $\sqrt[3]{6x^2}$

c. $\frac{1}{\sqrt[4]{w}}$

d. $\sqrt[3]{(6x)^2}$

4. (We do) Combine the radical expressions.

a. $\frac{\sqrt[4]{x^3}}{\sqrt[8]{x^2}}$

b. $\frac{\sqrt{x^3}}{\sqrt[3]{x^2}}$

c. $\sqrt{3}(\sqrt[4]{3})$

5. (They do) Simplify.

a. $(-8x^{15})^{-\frac{1}{3}}$

b. $(16x^{-8})^{-\frac{3}{4}}$

6. (They do) Simplify.

a. $16^{-2.5}$

b. $32^{-\frac{3}{5}}$

You do: Practice 6-4: Complete your assignment on a separate sheet of paper. Show all work.

1. Simplify each expression.

a. $16^{\frac{1}{4}}$

b. $(-3)^{\frac{1}{3}} \cdot (-3)^{\frac{1}{3}} \cdot (-3)^{\frac{1}{3}}$

c. $5^{\frac{1}{2}} \cdot 45^{\frac{1}{2}}$

2. Write each expression in radical form.

a. $x^{\frac{1}{4}}$

b. $x^{\frac{4}{5}}$

c. $x^{\frac{2}{9}}$

3. Write each expression in exponential form.

a. $\sqrt[3]{2}$

b. $\sqrt[3]{2x^2}$

c. $\sqrt[3]{(2x)^2}$

4. Simplify each number.

a. $(-216)^{\frac{1}{3}}$

b. $(\sqrt[4]{6})(\sqrt[3]{6})$

c. $32^{-0.4}$

5. Bone loss for astronauts may be prevented with an apparatus that rotates to simulate gravity. In the formula $N = \frac{a^{0.5}}{2\pi r^{0.5}}$, N is the rate of rotation in revolutions per second, a is the simulated acceleration in m/s^2 , and r is the radius of the apparatus in meters. How fast would an apparatus with a radius of 1.7 m have to rotate to simulate the acceleration of 9.8 m/s^2 that is due to Earth's gravity?

6-5 Solving Square Root and Other Radical Equations Part 1 Standards

A2.A.REI.D.6 (formerly A-REI.D.11) Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the approximate solutions using technology.

A2.A.REI.A.1 (formerly A-REI. A.1) Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A2.A.REI.A.2 (formerly A-REI. A.2) Solve rational and radical equations in one variable and identify extraneous solutions when they exist.

A2.A.CED.A.1 (formerly A-CED.A.1) Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and rational and exponential functions.

A2.A.CED.A.2 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Key Concepts

_____ - an equation that has a variable in a radicand or has a variable with a rational exponent.

It is possible to get extraneous solutions for square root and other radical equations, so

_____.

Examples

(I do) Solve.

1. $-10 + \sqrt{2x + 1} = -5$ 2. $3(x + 1)^{\frac{3}{5}} = 24$ 3. $3(x + 1)^{\frac{2}{3}} = 12$

6. (They do) For the meteor crater in Arizona, the formula $d = 2\sqrt[3]{\frac{V}{0.3}}$ relates the diameter d of the rim (in meters) to the volume V (in cubic meters). What is the volume of the Meteor Crater if the diameter is 1.2 m?

6-5 Solving Square Root and Other Radical Equations Part 2

Standards

A2.A.REI.D.6 (formerly A-REI.D.11) Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the approximate solutions using technology.

A2.A.REI.A.1 (formerly A-REI. A.1) Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A2.A.REI.A.2 (formerly A-REI. A.2) Solve rational and radical equations in one variable and identify extraneous solutions when they exist.

A2.A.CED.A.1 (formerly A-CED.A.1) Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and rational and exponential functions.

A2.A.CED.A.2 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

(We do) Solve. Check for extraneous solutions.

4. $\sqrt{x+2} - 3 = 2x$

5. $\sqrt{2x+1} - \sqrt{x} = 1$

You do: Practice 6-5: Complete your assignment on a separate sheet of paper. Show all work.

1. Solve.

a. $\sqrt{x+2}-2=0$

b. $\sqrt{2x+3}-7=0$

c. $2+\sqrt{3x-2}=6$

2. Solve.

a. $2(x-2)^{\frac{2}{3}}=50$

b. $(6x-5)^{\frac{1}{3}}+3=-2$

3. Solve. Check for extraneous solutions.

a. $\sqrt{4x+5}=x+2$

b. $\sqrt{-3x-5}-3=x$

4. A cylindrical can holds 28 in.³ of soup. If the can is 4 in. tall, what is the radius of the can to the nearest tenth of an inch? (*Hint*: $V = \pi r^2 h$)

5. Reasoning. If you are solving $4(x+3)^{\frac{3}{4}}=7$, do you need to use the absolute value to solve for x ? Why or why not?

6. Find the solutions of $\sqrt{x+2}=x$, are there any extraneous solutions? How do you know?

6-6 Function Operations

Standards

A2.A.CED.A.1 (formerly A-CED.A.1) Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and rational and exponential functions.

A2.A.CED.A.2 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Key Concepts

Addition	$(f + g)(x) = f(x) + g(x)$
Multiplication	$(f \cdot g)(x) = f(x) \cdot g(x)$
Subtraction	$(f - g)(x) = f(x) - g(x)$
Division	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$
Composition	$(g \circ f)(x) = g(f(x))$

Examples

- (I do) Let $f(x) = -2x + 6$ and $g(x) = 5x - 7$
 - Find $(f + g)(x)$. State the domain.
 - Find $(f - g)(x)$. State the domain.

- (We do) Let $f(x) = x^2 + 1$ and $g(x) = x^4 - 1$
 - Find $f \cdot g$. State the domain.
 - Find $\frac{f}{g}$. State the domain.

- (I do) Let $f(x) = x^3$ and $g(x) = x^2 + 7$
 - Find $(f \circ g)(x)$
 - Find $(g \circ f)(x)$
 - Find $(g \circ f)(2)$

4. (They do) A store offers a 20% discount on all items. You have a coupon worth \$5.
 - a. Write a function (f) to model discounting an item by 20%.
 - b. Write a function (g) to model applying a \$5 coupon.
 - c. Use a composition of your two functions to model how much you would pay for an item if the clerk applies the discount first and then the coupon.
 - d. Use a composition of your two functions to model how much you would pay for an item if the clerk applies the coupon first and then the discount.
 - e. Which composition gives the better discount? By how much?
 - f. Evaluate each composition function for an item that costs \$25.

You do: Practice 6-6: Complete your assignment on a separate sheet of paper. Show all work.

1. Let $f(x) = x - 2$ and $g(x) = x^2 - 3x + 2$. Find each of the following and state the domain.
 - a. Find $f + g$
 - b. Find $f - g$
 - c. Find $f \cdot g$
 - d. Find $\frac{f}{g}$
 - e. Find $f \circ g$
 - f. Find $g \circ f$
2. A car dealer offers a 15% discount of the list price x of any car on the lot. At the same time, the manufacturer offers a \$1000 rebate for each purchase of a car.
 - a. Write a function $f(x)$ to represent the price after discount.
 - b. Write a function $g(x)$ to represent the price after the \$1000 rebate.
 - c. Suppose the list price of a car is \$18,000. Use a composite function to find the price of the car if the discount is applied before the rebate.
 - d. Suppose the list price of a car is \$18,000. Use a composite function to find the price of the car if the discount is applied after the rebate.
 - e. **Reasoning** Between parts (c) and (d), will the dealer want to apply the discount before or after the rebate? Why?

6-7 Inverse Functions and Relations

Standards

A2. F.BF.B.4a (formerly F-BF.B.4) Find inverse functions. a. Find the inverse of a function when the given function is one-to-one.

A2. F.BF.A.1 (formerly F-BF.A.1) Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

A2.F.IF.B.3 Graph functions expressed symbolically and show key features of the graph, by hand and using technology.

Key Concepts

_____ - “undoes” the relation and maps b back to a .

_____ - if you have f as a function then f^{-1} is its inverse.

If f and f^{-1} are inverses, then $(f \circ f^{-1})(x) = x = (f^{-1} \circ f)(x)$

1. (I do) What is the inverse relation for each?

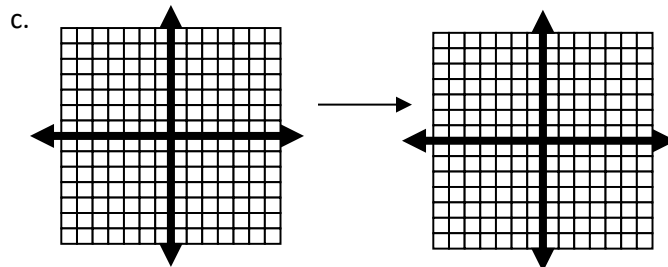
a. Relation s : $\{(5, 2), (0, 1), (-4, 3)\}$

b.

x	y
0	-1
2	0
3	2
4	3

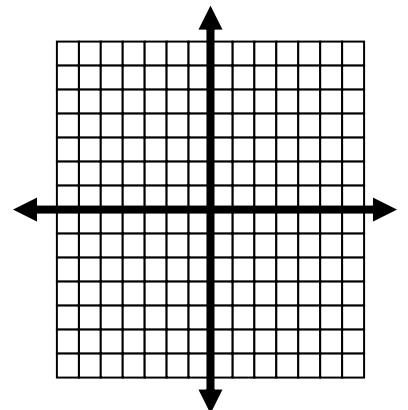
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x	y



2. (I do) Find the inverse of $y = x^2 - 1$.

a. Graph y and its inverse.



- b. State the domain of both y and its inverse.
- c. Is y^{-1} a function? Explain
3. (We do) Consider the function $f(x) = 6 - 4x$.
- a. State the domain of $f(x)$.
- b. Find f^{-1} and state its domain.
- c. Is f^{-1} a function?
4. (We do) Consider the function $f(x) = \sqrt{x - 2}$.
- a. State the domain of $f(x)$.
- b. Find f^{-1} and state its domain.
- c. Is f^{-1} a function?

5. If $f(x) = \frac{1}{2}x + 5$, find $(f \circ f^{-1})(652)$ and $(f^{-1} \circ f)(\sqrt{86})$.

6. (They do) The function $d = 4.9t^2$ represents the distance d in meters an object falls in t seconds due to the earth's gravity. Find the inverse of this function. How long does it take for a cliff diver to reach the water below if he is at a height of 24 meters?

You do: Practice 6-7: Complete your assignment on a separate sheet of paper. Show all work.

1. Find the inverse of the relation. $A = \{(2, -3), (0, -1), (-5, 2), (-3, 2)\}$.
2. Find the inverse of each function. Is the inverse a function? Graph and state the domain and range of each function and its' inverse
 - a. $y = \frac{x}{2}$
 - b. $y = x^2 + 4$
 - c. $y = (3x - 4)^2$
3. The formula for the area of a circle is $A = \pi r^2$.
 - a. Find the inverse of the formula. Is the inverse a function?
 - b. Use the inverse to find the radius of a circle that has an area of 82 in.².

6-8 Graphing Radical Functions

Standards

A2. F.BF.B.4a (formerly F-BF.B.4) Find inverse functions. a. Find the inverse of a function when the given function is one-to-one.

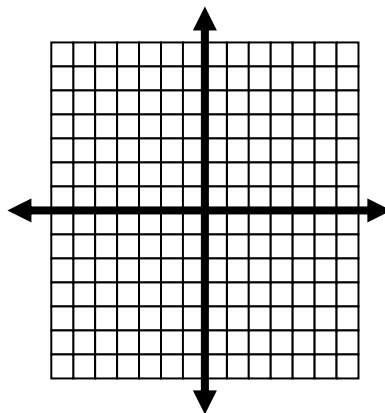
A2. F.BF.A.1 (formerly F-BF.A.1) Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

A2.F.IF.B.3 Graph functions expressed symbolically and show key features of the graph, by hand and using technology.

Examples

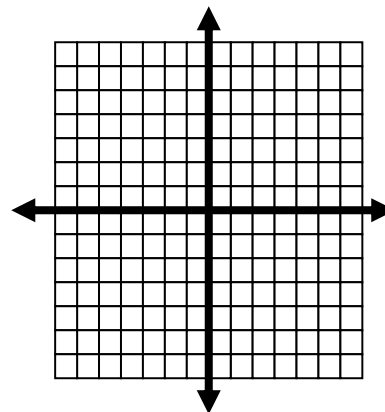
1. (I do) Graph $y = \sqrt{x}$

x	y

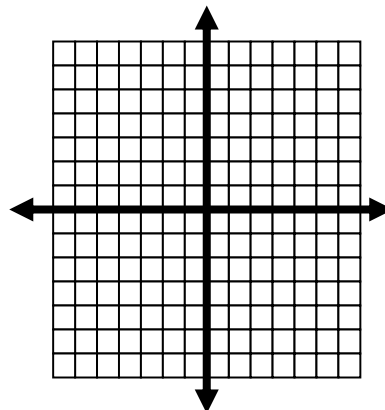


2. (We do) Graph the following functions

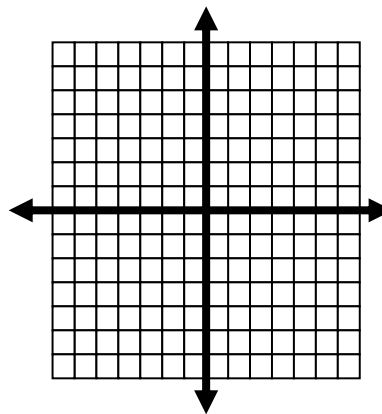
a. $y = \sqrt{x} - 2$ and $y = \sqrt{x} + 1$



b. $y = \sqrt{x+4} - 1$ and $y = \sqrt{x-1}$

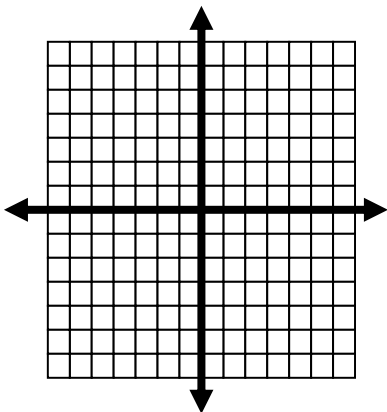


c. $y = -\frac{1}{2}\sqrt{x-3} + 1$ and $y = 2\sqrt[3]{x+1} - 4$

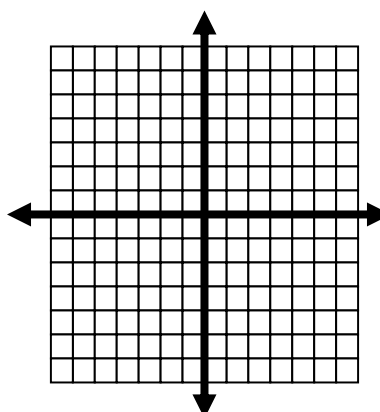


(They do)

3. Solve by graphing $\sqrt{x-3} = 2$



4. Solve by graphing $\sqrt{x+4} = -3$



You do: Practice 6-8: Complete your assignment on a separate sheet of paper. Show work.

1. Graph each function.

a. $y = \sqrt{x+3}$

b. $y = \sqrt{x-4}$

c. $y = -2\sqrt{x+1}$

2. Solve each square root equation by graphing. Round the answer to the nearest hundredth, if necessary. If there is no solution, explain why.

a. $\sqrt{x+2} = 7$

b. $\sqrt{4x+1} = 5$

c. $3\sqrt{3-x} = 10$

3. A periscope on a submarine is at a height h , in feet, above the surface of the water. The greatest distance d , in miles, that can be seen from the periscope on a clear day is given by $d = \sqrt{\frac{3h}{2}}$.

a. If a ship is 3 miles from the submarine, at what height above the water would the submarine have to raise its periscope in order to see the ship?

b. If a ship is 1.5 miles from the submarine, to what height would it have to be raised?