7-1 Exploring Exponential Models

Standards

A2. F.LE.A.1 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or input-output pairs.
A2. F.LE.B.3 Interpret the parameters in a linear or exponential function in terms of a context.
A2.F.IF.B.3 Graph functions expressed symbolically and show key features of the graph, by hand and using technology. a. Graph square root, cube root, and piecewise defined functions, including step functions and absolute value functions. c. Graph exponential and logarithmic functions, showing intercepts and end behavior.
A2.F.IF.B.5 Compare properties of two functions each represented in a different way
A2. F.IF.A.2 Calculate and interpret the average rate of change of a function (presented symbolically) Estimate the rate of change from a graph.
A2.A.REI.D.6 Explain why the x-coordinates of the points where the graphs of the equations y = f(*x*) and y = g(*x*) intersect are the solutions of the equation f(*x*) = g(*x*); find the approximate solutions using technology. Include cases where f(*x*) and/or g(*x*) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
A2.F.BF.B.3 Identify the effect on the graph of replacing f(*x*) by f(*x*) + *k*, *k* f(*x*), f(*kx*), and f(*x* + *k*) for specific values of k; find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Key Concepts

Characteristics	$y = ab^x$	$y = ab^{x-h} + k$
Asymptote		
Domain		
Range		

____- a function with the general form $y = ab^x$, where *x* is a real number, a ≠ 0, **b = 1 + r**, b > 0, and b ≠ 0.

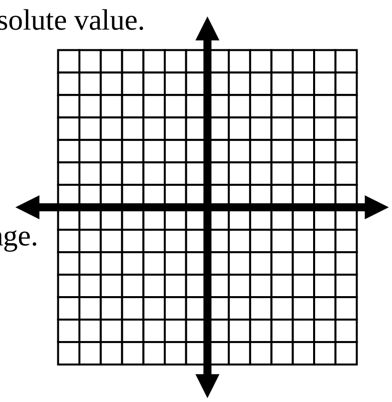
A __________________________is when b > 1 and a _________________________ is when 0 < b < 1.

_________________________- a line that a graph approaches as *x* or *y* increases in absolute value.

_________________________-model for exponential growth and decay.

Examples

1. (I do) Graph $y = 3(2)^x$. Identify the y-intercept, asymptote, domain and range.



2. Without graphing, determine whether the functions represent exponential growth or decay.
   a. (I do) $y = 3(\frac{2}{3})^x$
   b. (We do) $y = 0.25(2)^x$
   c. (We do) You invest $1000 into a college savings account for 4 years, the account pays 5% interest annually.

3. (They do) You invested $1000 in a savings account at the end of 6^{th} grade. The account pays 5% annual interest. How much money will be in the account at the end of 12^{th} grade?

4. (They do) The population of the Iberian lynx is 150 in 2003 and is 120 in 2004. If this trend continues and the population is decreasing exponentially, how many Iberian lynx will there be in 2014?

(You do) Practice 7-1: Complete your assignment on a separate sheet of paper. Show all work.

1. Graph $y = 0.75(4)^x$. State the y-intercept, asymptote, domain and range.
2. Explain how you know whether a function of the form $y = ab^x$ is exponential growth or decay?
3. Without graphing, determine whether the function represents exponential growth or decay. Then state the y-intercept.
   a. $y = 10(0.45)^x$ b. $y = 2(3)^x$
4. Identify each equation as linear, quadratic, or exponential.
   a. $y = 3(x + 1)^2$ b. $y = 4(3)^x$ c. $y = 2x + 5$ d. $y = 4(0.2)^x$
5. The population of Bainsville is 2000. The population is supposed to grow by 10% each year for the next 5 years. How many people will live in Bainsville in 5 years?
6. A music store sold 200 guitars in 2007. The store sold 180 guitars in 2008. The number of guitars that the store sells is decreasing exponentially. If this trend continues, how many guitars will the store sell in 2012?

7-2 Properties of Exponential Functions

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Key Concepts

Transformations of $y = ab^{x-h} + k$	
Vertical Translation (k units) *Up:* $y = b^x + k$ *Down:* $y = b^x - k$	**Horizontal Translation (h units)** *Right:* $y = b^{x-h}$ *Left:* $y = b^{x+h}$
Vertical Stretch/ Compression *Stretch (a>1):* $y = ab^x$ *Compression (0<a<1):* $y = ab^x$	**Reflection** *x-axis:* $y = -b^x$

*Remember____

_______- an irrational number approximately equal to 2.71828 and is useful for describing continuous growth or decay.

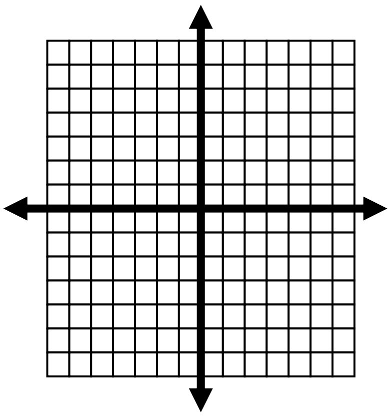
___________________________-continuously compounded interest formula.

Examples

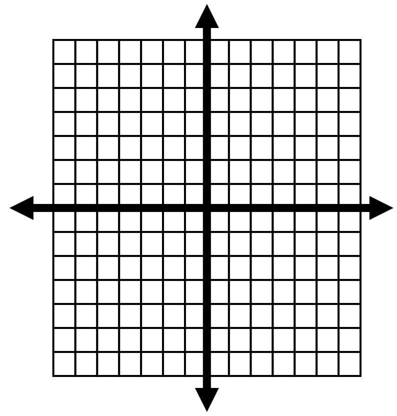
1. (I do) Suppose you invest $1000 at an annual interest rate of 4.8% compounded continuously. How much will you have in the account after 3 years?

2. (I do) Graph both on the same axis. Label the asymptote, state the domain, range and describe the transformation.

   a. $y = 3(2)^x$ and $y = -3(2)^x$

   b. $y = 6(\frac{1}{2})^x$ and $y = 6(\frac{1}{2})^{x-3} - 2$.

3. (They do) The best temperature to brew coffee is between 195°F and 205°F. Coffee is cool enough to drink at 185°F. The table shows temperatures from a sample cup of coffee. How long does it take for a cup of coffee to be cool enough to drink? Use an exponential model.

Time(min)	**Temp(°F)**
0	203
5	177
10	153
15	137
20	121
25	111
30	104

4. (They do) The half-life of Phosphorus-32 is 14.3 days. Write an exponential decay function for a 50-mg sample. Find the amount remaining after 84 days.

(You do) Practice 7-2: Complete your assignment on a separate sheet of paper. Show all work.

1. Graph $y = -2(3)^{x-1} + 4$. Label the asymptote and describe the transformation from $y = 2(3)^x$

2. Is investing $2000 in an account that pays 5% annual interest the same as investing $2000 at 5% compounded continuously? Explain.

3. Evaluate e^4

4. Archaeologists use carbon-14 with a half-life of 5730 years to determine the age of artifacts in carbon dating. Write an exponential decay function for a 24-mg sample. How much carbon -14 remains after 1000 years?

7-3 Logarithmic Functions as Inverses

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Key Concepts

______________________- has base *b* of a positive number *y* is defined as follows: If ____________, then _______________.

________________________________- a logarithm that uses base 10.

______________________________________- the inverse of an exponential function.

Characteristics	$y = \log_b x$	$y = \log_b(x - h) + k$
Asymptote		
Domain		
Range		

Examples

1. (I do) Write in logarithmic form.

a. $4^{-3} = \frac{1}{64}$ b. $6^2 = 36$ c. $17^0 = 1$

2. (I do) Write in exponential form.

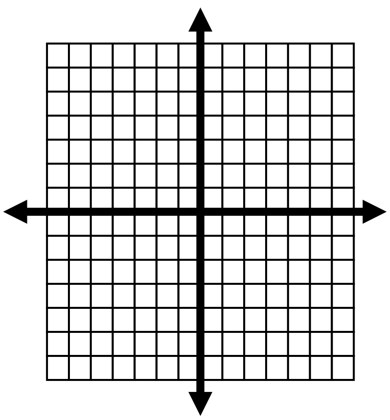
   a. $\log_2 8 = 3$ b. $\log 1000 = 3$ c. $\log_8 \frac{1}{4} = -\frac{2}{3}$

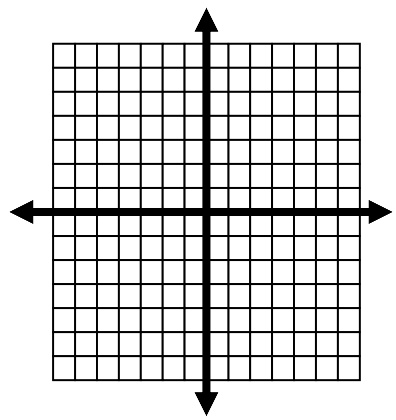
3. (We do) Evaluate.

   a. $\log_3 81$ b. $\log_5 125$ c. $\log_8 2$

4. (They do) Graph. State the asymptote, domain and range. Then describe the transformation.

   a. $y = \log_4 x$ b. $y = \log_5(x-1)+2$





5. (They do) The formula $\log\frac{I_1}{I_2} = M_1 - M_2$ compares the intensity levels of earthquakes where I is the intensity level determined by a seismograph M is the magnitude. How many times more intense is a 9.3 earthquake than an 8.7 earthquake?

(You do) Practice 7-3: Complete your assignment on a separate sheet of paper. Show all work.

1. Write in logarithmic form.

   a. $25 = 5^2$ b. $4 = (\frac{1}{2})^{-2}$ c. $\frac{1}{10} = 10^{-1}$

2. Evaluate.

   a. $\log_7 49$ b. $\log_2 \frac{1}{4}$ c. $\log_9 9$

3. Graph $y = 2^x$ and $y = \log_2 x$ on the same axis. State the asymptote, domain, range and describe the transformation.

4. Compare the intensity level of a 7.9 earthquake and a 3.2 earthquake.

5. Error Analysis. Describe the error in the problem, then fix it.

$$x = \log_{27} 3$$
$$x^3 = 27$$
$$\sqrt[3]{x^3} = \sqrt[3]{27}$$
$$x = 3$$

7-4 Properties of Logarithms

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Key Concepts

Properties of Logarithms

Product Property: $\log_b MN = \log_b M + \log_b N$

Quotient Property: $\log_b \frac{M}{N} = \log_b M - \log_b N$

Power Property: $\log_b M^n = n \log_b M$

Change of Base Formula: $\log_b M = \frac{\log_c M}{\log_c b}$

Examples

1. (I do) Write each logarithmic expression as a single logarithm.
   a. $\log_4 5x + \log_4 3x$ b. $6\log_5 x + \log_5 y$ c. $\log_4 64 - \log_4 16 + \log_4 4$

2. (I do) Expand each logarithm.
   a. $\log_7(\frac{x}{y})$ b. $\log(4p^3)$ c. $\log_9 \frac{x^4}{729}$

3. (We do) Use the Change of Base Formula to evaluate $\log_6 12$.

4. (They do) Determine if each statement is true or false.

   a. $\log_2 4 + \log_2 8 = 5$ b. $\log(x - 2) = \frac{\log x}{\log 2}$

5. (They do) The pH of a substance equals $-\log\ [H^+]$, where $[H^+]$ is the concentration of hydrogen ions. $[H^+]$ for household ammonia is 10^{-11} . $[H^+]$ for vinegar is $6.3\ \times\ 10^{-3}$. What is the difference of the pH levels of ammonia and vinegar?

(You do) Practice 7-4: Complete your assignment on a separate sheet of paper. Show all work.

1. Write each expression as a single logarithm.

   a. $\log_4 2 + \log_4 8$ b. $\log 8 - 2\log 6 + \log 3$ c. $\log_7 x + \log_7 y - 2\log_7 z$

2. Expand each logarithm.

   a. $\log_5 x^2 y^3$ b. $\log_2 \sqrt{x}$ c. $\log_5 \frac{25}{x}$ d. $\log 10 m^4 n^{-2}$

3. Determine whether the statement is true or false.

   a. $\log_4 7 - \log_4 3 = \log_4 4$ b. $\log_3 \frac{3}{2} = \frac{1}{2} \log_3 3$

4. The pH of a substance equals $-\log\ [H^+]$, where $[H^+]$ is the concentration of hydrogen ions. $[H^+]$ for dish detergent is 10^{-12}. What is the pH level of dish detergent?

7-5 Exponential and Logarithmic Equations

Standards

A2.A.CED.A.1 Create equations and inequalities in one variable and use them to solve problems.
A2. F.IF.B.4. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the properties of exponents to interpret expressions for exponential functions.
A2. N.Q.A.1 Identify, interpret, and justify appropriate quantities for the purpose of descriptive modeling.

Key Concepts

________________________________- an equation of the form $b^{cx} = a$, where the exponent includes a variable.
_________________________- an equation that includes a logarithmic expression.

Steps to Solving Exponential Equations

1.
2.
3.
4.

Steps to Solving Logarithmic Equations

1.
2.
3.

Examples

1. (We do) Solve
   a. $16^{3x} = 8$
   b. $12^{x-2} = 20$
   c. $7 - 5^{2x-1} = 4$

2. (We do) Solve
   a. $\log_3(2x - 2) = 4$
   b. $3\log x - \log 2 = 5$

3. (They do) Solve $\log(x-3)+\log x=1$

4. (They do) As a town gets smaller, the population of its high school decreases by 6% each year. The senior class has 160 students now. In how many years will it have about 100 students? Write an equation. Then solve.

(You do) Practice 7-5: Complete your assignment on a separate sheet of paper. Show all work.

1. Solve each equation

a. $3^x = 9$ b. $2^{x+1} = 25$ c. $\log 4x = 2$ d. $\log x - \log 2 = 3$

2. Describe and correct the error made in solving the equation.

$$\log_2 x = 2\log_3 9$$
$$\log_2 x = \log_2 9^2$$
$$x = 9^2$$
$$x = 81$$

3. Suppose \$1000 is deposited at an interest rate of 5% per year *x* months after the money is deposited.

a. Write an exponential function that models this situation.

b. Use a graphing calculator to predict how many months it will be until the account is worth \$1100.

c. How many years will it be until the account reaches \$5000? Solve algebraically.

7-6 Natural Logarithms

Standards

A2.A.REI.D.6 Explain why the *x*-coordinates of the points where the graphs of the equations *y* = *f(x)* and *y* = *g(x)* intersect are the solutions of *f(x)* = *g(x)*; find the appropriate solutions using technology.
A2.F.LE.A.2 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
A2.F.IF.A.1 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
A2.F.IF.A.2 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
A2.F.IF.B.3 Graph functions expressed symbolically and show key features of the graph, by hand and using technology. Graph exponential and logarithmic functions, showing intercepts and end behavior.

Key Concepts

____or $\ln x$ is equal to $\log_e x$ and $\ln x = y$ if and only if $e^y = x$

Examples

1. (I do) Write each expression as a single logarithm.

   a. $2\ln 15 - \ln 75$ b. $3\ln x - 2\ln 2x$ c. $\frac{1}{3}(\ln x + \ln y) - 4\ln z$

2. (We do) Expand each logarithmic expression.

   a. $\ln \frac{x\sqrt{y}}{2}$

3. (We do) Solve the natural logarithmic equation.

   a. $\ln\ (x-3)^2 = 4.$ b. $3 - 4\ln(8x+1) = 12$

4. (They do) Solve the exponential equation.

   a. $4e^{2x} + 2 = 16$

5. (They do) A spacecraft can attain a stable orbit 300 km above Earth if it reaches a velocity of 7.7 km/s. The formula for a rocket's maximum velocity *v* in km/s is $v = -0.0098t + c \ln R$. The booster rocket fires for *t* seconds and the velocity of the exhaust is *c* km/s. *R* is the ratio of the mass of the rocket filled with fuel to its mass without fuel. Suppose a rocket has a mass ratio of 25, a firing time of 100 s and an exhaust velocity of 2.8 km/s. Can the spacecraft attain a stable orbit of 300 km above earth?

(You do) Practice 7-6: Complete your assignment on a separate sheet of paper. Show all work.

1. Write the expression as a single logarithm.

   a. $\ln 18 - \ln 10$ b. $\frac{1}{2}\ln x - \ln y$

2. Solve

   a. $\ln 3x = 6$ b. $\ln(4x - 1) = 3$ c. $e^{3x} + 5 = 6$

3. The formula $P = 50e^{-\frac{t}{250}}$ gives the power output *P*, in watts, needed to run a certain satellite for *t* days. Find how long a satellite with a 12 W power output will operate.