8-1 Inverse Variation

Standard

A2. F.BF.B.4 Find inverse functions. a. Find the inverse of a function when the given function is one-to-one

Objectives

Students will be able to recognize and use inverse variation including joint and combined variations.

Key Concepts

______________________________ - a function of the form $y = kx$, $k = \frac{y}{x}$, where $k \neq 0$

______________________________ - a function of the form $y = \frac{k}{x}$, $k = xy$, where $k \neq 0$

______________________________ – combines direct and inverse variations in more complicated relationships

______________________________ – one quantity varies directly with 2 or more quantities

Combined Variation	Equation Form
z varies jointly with *x* and *y*	
z varies jointly with *x* and *y* and inversely with *w*	
z varies directly with *x* and inversely with the product *wy*	

Examples

1. (I do) Is the relationship between the variables a direct variation, an inverse variation, or neither? Write function models for direct and inverse variation.

a.

x	*y*
2	0.7
4	0.35
7	0.2
14	0.1

b.

x	*y*
-2	6
-1.3	5
7	0.4
10	-15

c.

x	*y*
-2	5
4	-10
6	-15
8	-20

2. (I do) Suppose that x and y vary inversely, and $x = 2$ when $y = 8$. Write the function that models the inverse variation. Find y when $x = 4$.

3. (We do) Z varies directly with x and inversely with y. When $x = 8$ and $y = 2$, $z = 12$. Write the function that models the variation. Find z when $x = 4$ and $y = 6$.

4. (We do) Z varies jointly with x and y. When $x = 5$ and $y = 3$, $z = 60$. Write the function that models the variation. Find z when $x = 5$ and $y = 0.25$.

5. (They do) DECA has decided to pick up litter each weekend at the park. Each week there is approximately the same amount of litter. The table shows the number of students who worked each of the first four weeks and the time needed.

# of students (*n*)	3	5	12	17
Time (*t*)	85	51	21	15

a. Does the function model direct or inverse variation? How do you know?

b. What function models the data?

c. How many students should there be to complete the pick up in at most 30 minutes each week?

You do: Practice 8-1: Complete your assignment on a separate sheet of paper. Show all work.

1. Is the relationship between the variables a direct variation, an inverse variation, or neither? Write function models for direct and indirect variation.

a.

x	1	3	12	15
y	6	2	0.5	0.4

b.

x	-3	5	6	16
y	-15	25	30	80

2. Describe the variation in the given equation $p = \frac{kqrt}{s}$
3. Suppose that *x* and *y* vary inversely. Write a function that models the variation and find *y* when *x* = 10.
   a. *x* = -13, when *y* = 100
   b. *x* = 20, when *y* = -4
4. In a bake sale, you recorded the number of muffins sold and the amount of sales in the table.
   a. What is a function that relates the sales and the number of muffins?
   b. How many muffins would you have to sell to make at least $250 in sales?

# of muffins (*m*)	5	8	13	20
Sales (*s*)	$12.50	$20.00	$32.50	$50.00

8-2 The Reciprocal Function Family

Standards

A2. F.BF.B.4 Find inverse functions. a. Find the inverse of a function when the given function is 1-1.
A2. F.IF.A.1 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
A2. F.IF.B.3 Graph functions expressed symbolically and show key features of the graph, by hand and using technology.
A2. N.Q.A.1 Identify, interpret, and justify appropriate quantities for the purpose of descriptive modeling.

Objective

Students will be able to graph reciprocal functions and interpret key features.

Key Concepts

________________________________ – belongs to the family whose parent is $f(x) = \frac{1}{x}$, where $x \neq 0$.

_____________________– each part of the graph of a reciprocal function.

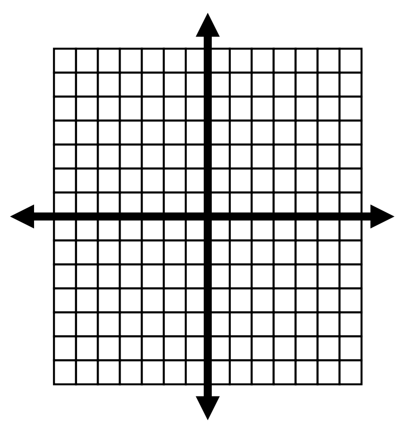
The line ________________ is a **vertical asymptote** and is a horizontal translation.

The line _____________ is a **horizontal asymptote** and is a vertical translation.

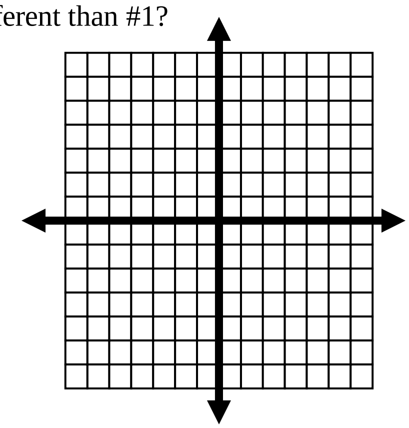
________________________________- general form for a reciprocal function.

Examples

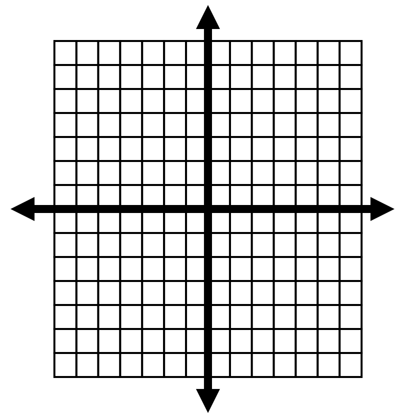
1. (I do) Graph $y = \frac{1}{x}$



2. (We do) Graph $y = \frac{6}{x}$. Identify the *x*- and *y*- intercepts and the asymptotes of the graph. Also, state the domain and range. How is this graph different than #1?



3. (We do) What is the graph of $y = \frac{1}{x-3} + 1$.



   a. State the asymptotes, domain and range.

   b. Describe the transformation.

4. (They do) Write an equation for the translation of $y = \frac{3}{x}$ that has asymptotes at $x = -1$ and $y = -7$

5. (They do) The rowing club is renting a 57- passenger bus for a day trip. The cost of the bus is \$750. Five passengers will be chaperones. If the students who attend share the cost of the bus equally, what function models the cost ***C*** with respect to ***x***, the number of students who attend? What is the domain? How many students must ride the bus to make the cost per students no more than \$20?

You do: Practice 8-2: Complete your assignment on a separate sheet of paper. Show all work.

1. Graph $y = \frac{3}{x}$

2. Describe the transformation from the graph of $y = \frac{1}{x}$ to the following.

   a. $y = \frac{1}{x} + 5$ b. $y = -\frac{4}{x}$

3. Consider $y = \frac{5}{x+2} - 7$. State the transformations, domain, range and asymptotes.

4. Write an equation for the translation of $y = \frac{2}{x}$ with asymptotes at $x = -2$ and $y = 3$.

5. The weight P in pounds that a beam can safely carry is inversely proportional to the distance *D* in feet between the supports of the beam. For a certain type of wooden beam, $P = \frac{9200}{D}$. What distance between supports is needed to carry 1200 lb?

8-3 Rational Functions and Their Graphs

Standards

A2. F.IF.A.1 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
A2. F.IF.B.3 Graph functions expressed symbolically and show key features of the graph, by hand and using technology

Objective

Students will graph rational functions, find points of discontinuity and asymptotes

Key Concepts

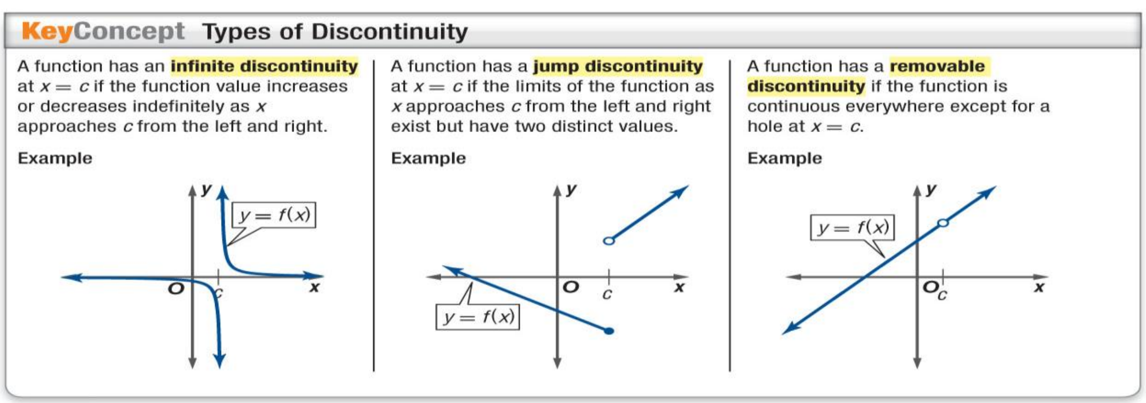
__________________ – a function that you can write in the form $f(x) = \frac{P(x)}{Q(x)}$, where $Q(x) \neq 0$

_____________________________ – a graph that has no breaks, jumps, or holes.

_________________________ – a graph that has jumps, breaks or holes.

___________________________ – the point at which the graph is not continuous ($x = a$)

_______________________________ – the graph has a vertical asymptote at $x = a$ if it has non-removable discontinuity at $x = a$.



___________________________ **-** to find a horizontal asymptote, compare the degree of the numerator to the degree of the denominator.

- If degree of numerator < degree of denominator, then the horizontal asymptote is $y = 0$
- If degree of numerator = degree of denominator, then the horizontal asymptote is y = ratio of leading coefficients.
- If degree of numerator > degree of denominator, then there is no horizontal asymptote.

Examples

1. (I do) Consider the rational function $y = \frac{x+4}{x^2-x-12}$
   a. What is the domain of the rational function?

   b. Identify the points of discontinuity. Are the points of discontinuity removable or non-removable?

   c. What are the *x*- and *y* – intercepts?

2. (We do) Consider the rational function $y = \frac{2x}{x^2+12}$
   a. What is the domain of the rational function?

   b. Identify the points of discontinuity. Are the points of discontinuity removable or non-removable?

   c. What are the *x*- and *y* – intercepts?

3. (They do) Consider the rational function $y = \frac{x^2-4}{x+2}$
   a. What is the domain of the rational function?

   b. Identify the points of discontinuity. Are the points of discontinuity removable or non-removable?

   c. What are the *x*- and *y* – intercepts?

4. (I do) What are the vertical asymptotes for the graph?
    a. $y = \frac{(x+3)}{(x-3)(x+2)}$

    b. $y = \frac{(x+7)}{(x^2+9x+14)}$

5. (We do) What are the horizontal asymptotes for the graph?
    a. $y = \frac{-4x+3}{2x+1}$

    b. $y = \frac{x-2}{x^2-2x-3}$

    c. $y = \frac{x^2}{4x-1}$

6. (They do) Graph the rational function $y = \frac{x+1}{x^2-x-6}$

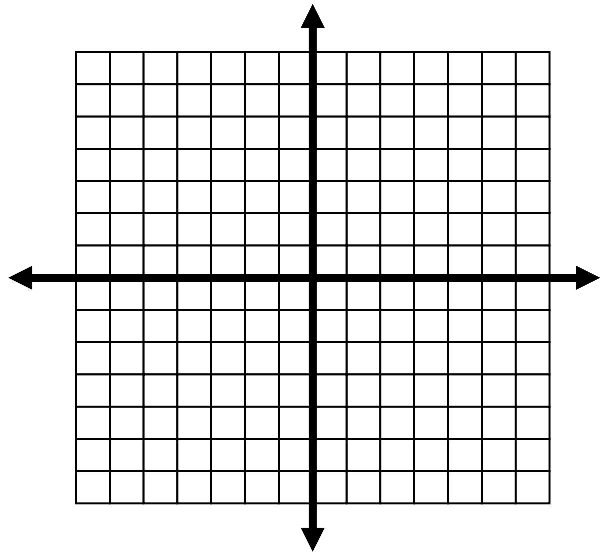
Step 1: Find HA

Step 2: Factor

Step 3: Find VA

Step 4: Find *x*- and *y*- intercepts

Step 5: Graph and get additional points on the graph



7. (They do) You work at a pharmacy that mixes different concentrations of saline. The pharmacy has a supply of two different concentrations, 0.5% and 2%. The function $y = \frac{100(0.02)+x(0.005)}{100+x}$ gives the concentration of the mixture after adding *x* milliliters of the 0.5% solution to 100 milliliters of the 2%. How many milliliters of the 0.5% solution must you add for the combined solution to have a concentration of 0.9%?

(You do) Practice 8-3: Complete your assignment on a separate sheet of paper. Show all work.

1. State the domain, find any points of discontinuity for each rational function, state the *x*- and *y*-intercepts. Are there any vertical asymptotes? Are the points of discontinuity removable or non-removable?

a. $y = \frac{x+5}{x^2+9x+20}$ b. $y = \frac{x-1}{(x+1)^2}$ c. $y = \frac{x^2-x-2}{3x^2-7x+2}$

2. Find the horizontal asymptotes.

a. $y = \frac{x-3}{x+5}$ b. $y = \frac{x-3}{x^2+5x+6}$ c. $y = \frac{x^2-1}{2x+2}$

3. Sketch the graph of each rational function $y = \frac{x+3}{x^2-7x+6}$

4. (See example 7, use the same function) How many milliliters of the 0.5% solution must be added to the 2% solution to get a 0.65% solution?

8-4 Rational Expressions

Standards

A2.A.APR.C.4 Rewrite rational expressions in different forms.

Objective

Students will rewrite and simplify rational expressions.

Key Concepts

____ – the quotient of two polynomials.

___________________________ – the numerator and denominator of a rational expression have no common factor

Examples

1. (I do) Write the expression in simplest form. State any restrictions on the variable.

   a. $\frac{24x^2y}{-6x^2y^3}$ b. $\frac{12-4x}{x^2-9}$ c. $\frac{x^2-6x-16}{x^2+5x+6}$

2. (I do) What is the product $\frac{x^2-25}{x^2+4x+3} \cdot \frac{x^2+x-6}{x-5}$ in simplest form? State any restrictions on the variable.

3. (We do) What is the quotient $\frac{x^2+5x+4}{x^2+x-12} \div \frac{x^2-1}{2x^2-6x}$ in simplest form? State any restrictions on the variable.

4. (They do) Your community is building a park. It wants to fence in a play space for toddlers. It wants the maximum area for a given amount of fencing. One measure of efficiency in fencing is the ratio of the area to the perimeter. The most efficient use of fencing will have the greatest ratio.
    a. Which shape, a square or circle, provides a more efficient use of fencing?

    b. Does this hold true for a perimeter of 40 feet?

You do: Practice 8-4: Complete your assignment on a separate sheet of paper. Show all work.

1. Simplify each rational expression. State any restrictions on the variable.

a. $\frac{4x-12}{8x+24}$ b. $\frac{5x^2y}{15xy^2}$ c. $\frac{x^2+8x+16}{x^2-2x-24}$

2. Multiply or Divide. State any restrictions on the variable.

a. $\frac{x^2+3x-10}{x^2+4x-12} \cdot \frac{3x+18}{x+3}$ b. $\frac{x^2-7x+10}{x^2-8x+15} \div \frac{4-x^2}{x^2+3x-18}$

3. Is the equation $y = \frac{x+1}{x^2+1}$ in simplest form? Explain how you can tell.

4. A student claims that $x = 2$ is the only solution of the equation $\frac{x}{x-2} = \frac{2}{x-2}$. Is the student correct? Explain.

5. Write a rational expression that simplifies to $\frac{x}{x+1}$.

8-5 Adding and Subtracting Rational Expressions

Standard

A2.A.APR.C.4 Rewrite rational expressions in different forms.

Objective

Students will add, subtract and simplify rational expressions.

Key Concepts

______________________________ - a fraction that has a fraction in its numerator or denominator or in both its numerator and denominator.

Steps to add or subtract Rational Expressions

1.

2.

3.

4.

Examples

1. (I do) What is the least common multiple (LCM)?
   a. $2x^2 - 8x + 8$ and $15x^2 - 60$

   b. $12x^2y(x^2 + 2x + 1)$ and $18y^3(x^2 + 5x + 4)$

2. (We do) Add or Subtract. Write your answer in simplest form. State any restrictions on the variable.
   a. $\frac{1}{3x^2+21x+30} + \frac{4x}{3x+15}$

   b. $\frac{2x}{x^2-2x-3} - \frac{3}{4x+4}$

3. (We do) Simplify the complex fraction.

a. $\dfrac{\frac{1}{x}+\frac{1}{y}}{\frac{2}{y}-\frac{1}{x}}$

b. $\dfrac{-3}{\frac{5}{x}+y}$

You do:Practice 8-5: Complete your assignment on a separate sheet of paper. Show all work.

1. Add or subtract. State any restrictions on the variable.

a. $\frac{x+11}{3x-5}+\frac{x-21}{3x-5}$

b. $\frac{1}{x^2-4}+\frac{6}{x+2}$

c. $\frac{b-4}{b^2+2b-8}-\frac{b+2}{b^2-16}$

d. $\frac{2x}{x^2-x-2}-\frac{4x}{x^2-3x+2}$

2. Simplify the complex fraction.

a. $\dfrac{\frac{2}{x}}{\frac{3}{y}}$

b. $\dfrac{1+\frac{2}{x}}{4-\frac{6}{x}}$

3. A rectangle has a length of $\frac{10b}{6b-6}$ and a width of $\frac{b+2}{2b-2}$. Write an expression to represent the perimeter of the rectangle (P = 2L + 2W).

8-6 Solving Rational Equations

Standard

A2.A.REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. Tasks are limited to square root, cube root, polynomial, rational, and logarithmic functions.
A2.A.REI.A.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
A2.A.REI.D.6 Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the approximate solutions using technology. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Objective

Students will solve rational equations

Key Concepts

________________________________ – an equation that contains at least one rational expression.

Steps to Solve a Rational Equation:

1.
2.
3.
4.
5.

Examples

1. (I do) What are the solutions of the rational equation?
   a. $\frac{2}{3}+\frac{3x-1}{6}=\frac{5}{2}$
   b. $\frac{x}{x-3}+\frac{x}{x+3}=\frac{2}{x^2-9}$

2. (We do) What are the solutions of the rational equation? $\frac{x-1}{x^2+3x+2}+\frac{2x}{x+2}=\frac{x-1}{x+1}$

3. What are the solutions of the rational equation? Use a graphing calculator. $\frac{2}{x+2}+\frac{x}{x-2}=1$

4. A flight across the U.S takes longer east to west than it does west to east. Assume the winds are constant in the eastward direction. When flying westward, the headwind decreases the airplane's speed. When flying eastward, the tailwind increases its speed. The time for a 1850 mile round trip is $7\frac{3}{4}$ h. If the airplane cruises at 480 mi/h, what is the speed of the wind? ($d = rt$)

You do: Practice 8-6: Complete your assignment on a separate sheet of paper. Show all work.

1. Solve each equation. Check each solution.

   a. $\frac{4}{x-2}=\frac{x-1}{x-2}$ b. $\frac{2a+1}{6}+\frac{a}{2}=\frac{a-1}{3}$ c. $\frac{1}{x}+\frac{x}{2}=\frac{x+4}{2x}$

2. You are riding your bike to a store 4 mi away. When there is no wind, you ride at 10 mi/h. Today your trip took 1 hour. What was the speed of the wind today?