9-1 Mathematical Patterns

Objectives

Identify mathematical patterns found in a sequence. Use a formula to find the *n*th term of a sequence.

State Standards

A2. F.BF. A.1a Write a function that describes a relationship between two quantities.
A2. F.BF.A.2 Know and write arithmetic and geometric sequences with an explicit formula and use them to model situations.
A2. F.LE.A.1 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a table, a description of a relationship, or input-output pairs**.**

Key Concepts

___________________________________- an ordered list of numbers

____- each number in a sequence

____- describes the *n*th term of a sequence using the number *n*

____- relates each term after the first one to the one before it.

Examples

1. (I do) Consider the sequence 2, 4, 6, 8, 10,… Create a table for the find the *n*th term.

n	***n*th term**

2. (We do) A sequence has an explicit formula $a_n = 3n - 2$. What are the first 10 terms of the sequence?

3. (I do) What is a recursive formula for each sequence?
   a. 1, 4, 7, 10, 13, …

   b. 1, 2, 6, 24, 120, …

4. (We do) Write an explicit formula for each sequence. Find the tenth term.
   a. 3, 5, 7, 9, 11, …

   b. 2, 4, 8, 16, 32, …

   c. 1, 4, 9, 16, 25, …

5. (They do) Pierre began the year with an unpaid balance of $300 on his credit card. Pierre is charged 1.8% on any unpaid balance in addition to a $29 penalty for each month he fails to make a minimum payment.
   a. Write a recursive definition for this situation.

   b. What did Pierre owe after 4 months of nonpayment?

You do: Practice 9-1: Complete your assignment on a separate sheet of paper. Show all work.

1. Find the first 5 terms of each sequence.
   a. $a_n = 5n - 3$ b. $a_n = n^2 - 2n$

2. What is a recursive definition for the sequence 3, 6, 12, 24,…?

3. What is an explicit formula for the sequence 5, 8, 11, 14,…?

4. Explain the difference between an explicit formula and a recursive definition. Give an example of each.

5. A student claims that $a_n = 3n + 1$ is an explicit formula for the sequence 2, 5, 8, 11, 14,… Is the student correct? If not, correct the student's error and write a correct explicit formula for the sequence.

6. You walk 1 mile the first day of training, 1.2 miles the second day, 1.6 miles the third day, and 2.4 miles the fourth day. Determine how many miles you would walk on the seventh day.

9-2 Arithmetic Sequences

Objectives

Identify mathematical patterns found in a sequence. Use a formula to find the *n*th term of a sequence.

State Standards

A2. F.BF. A.1a Write a function that describes a relationship between two quantities.
A2. F.BF.A.2 Know and write arithmetic and geometric sequences with an explicit formula and use them to model situations.
A2. F.LE.A.1 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a table, a description of a relationship, or input-output pairs**.**

Key Concepts

____-a sequence where the difference between consecutive terms is constant.

____- the difference between 2 consecutive values in an arithmetic sequence.

______________-the arithmetic mean (average of 2 numbers)

take note

Key Concept Arithmetic Sequence

An arithmetic sequence with a starting value a and common difference d is a sequence of the form

$$a, a + d, a + 2d, a + 3d, \dots$$

A recursive definition for this sequence has two parts:

$a_1 = a$ initial condition

$a_n = a_{n-1} + d$, for $n > 1$ recursive formula

An explicit definition for this sequence is a single formula:

$a_n = a + (n - 1)d$, for $n \geq 1$

Examples

1. (I do) Is the sequence arithmetic? If so state a and d.
   a. 2, 5, 8, 11, 14, …

   b. 1, 4, 9, 16, 25, …

2. (I do) What is the 100th term of the sequence 6, 11, 16, …?

3. (I do) What is the missing term in the sequence 15, ___, 59, …?

4. (We do) What are the missing terms in the sequence 100, ___, ___, 82, …?

5. (We do) The arithmetic mean of the monthly salaries of two employees is $3210. One employee earns $3470 per month. What is the monthly salary of the other employee?

6. (They do) A student deposits the same amount of money into her bank account each week. At the end of the second week, she has $35 in her account. At the end of the third week she has $50 in her account. How much will she have in her bank account at the end of the ninth week?

You do: Practice 9-2: Complete your assignment on a separate sheet of paper. Show all work.

1. Write an explicit formula and find the tenth term of the sequence.
   a. 2, 8, 14, 20,… b. 15, 23, 31,…
2. Find the missing term(s) of the sequence.
   a. 4,___, 22,… b. …, 25,____, ____, 67,…
3. Give an example of a sequence that is not an arithmetic sequence.
4. A student claims that the next term of the arithmetic sequence 0, 2, 4, … is 8. Explain and correct the student's error.

9-3 Geometric Sequences

Objective

Students will define, identify and apply geometric sequences

State Standards

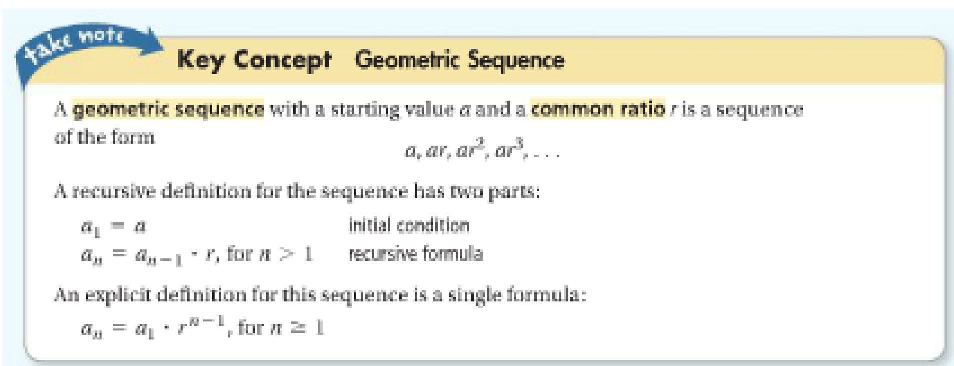
A2. F.BF. A.1a Write a function that describes a relationship between two quantities.
A2. F.BF.A.2 Know and write arithmetic and geometric sequences with an explicit formula and use them to model situations.
A2. F.LE.A.1 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a table, a description of a relationship, or input-output pairs**.**

Key Concepts

_______________________________________- a sequence of the form a, ar, ar^2, ar^3. It is formed by multiplying each term by a constant.

_______________________________________-the ratio of any term to its preceding term.

_______________________________________ -the geometric mean of two positive numbers *x* and *y*.


take note

Key Concept Geometric Sequence

A **geometric sequence** with a starting value a and a **common ratio** r is a sequence of the form

$$a, ar, ar^2, ar^3, \ldots$$

A recursive definition for the sequence has two parts:

$a_1 = a$ initial condition
$a_n = a_{n-1} \cdot r$, for $n > 1$ recursive formula

An explicit definition for this sequence is a single formula:

$a_n = a_1 \cdot r^{n-1}$, for $n \geq 1$

Examples

1. (I do) Is the sequence geometric? If so, what are a_1 and r
   a. 3, 6, 12, 24, 48,…

   b. 3, 6, 9, 12, 15, …

   c. $3^5, 3^{10}, 3^{15}, 3^{20}, \ldots$

2. (We do) Write the recursive and explicit definitions for the geometric sequence. Find the 10^{th} term
   a. 4, 12, 36, …

   b. 8, 4, 2, …

3. (We do) Use the geometric mean to find the missing term in the sequence 3, ___, 12, …

4. (They do) Find the missing terms in the sequence 2, ___, ___, -54,…

5. (They do) Suppose a balloon is filled with 5000 cm^3 of helium. It then loses one fourth of its helium each day. How much helium will be left in the balloon at the start of the tenth day?

You do Practice 9-3: Complete your assignment on a separate sheet of paper. Show all work.

1. Determine whether each sequence is geometric. If so, find the common ratio, write an explicit formula and find the eighth term.
   a. 5, 10, 15,… b. 10, 20, 40,… c. 1, -3, 9,… d. 1, 4, 9,…
2. Find the missing term in the geometric sequence.
   a. 4, _ __, 16,… b. 2, __ _, 50,…
3. Find the missing terms in the sequence 972, ___ , ___, ___, 12,…
4. Explain how you can determine whether a sequence is geometric or arithmetic.

9-4 Arithmetic Series

State Standards

Objective

Students will define, arithmetic series and find their sums.

State Standards

A2. F.BF. A.1a Write a function that describes a relationship between two quantities.
A2. F.BF.A.2 Know and write arithmetic and geometric sequences with an explicit formula and use them to model situations.
A2. F.LE.A.1 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a table, a description of a relationship, or input-output pairs**.**

Key Concepts

____- the indicated sum of the terms of a sequence

____- has a first and last term

____- continues without end

____- a series whose terms form an arithmetic sequence.

take note

Property Sum of a Finite Arithmetic Series

The sum S_n of a finite arithmetic series $a_1 + a_2 + a_3 + \cdots + a_n$ is

$$S_n = \frac{n}{2}(a_1 + a_n)$$

where a_1 is the first term, a_n is the nth term, and n is the number of terms.

Examples

1. (I do) Find the sum of the finite arithmetic series 7 + 8 + 9 + 10 + … + 15

2. (I do) What is the sum of the even numbers from 2 to 100?

3. (We do) What is summation notation for the series 7 + 11 + 15 + … + 207?

4. (We do) Expand and find the sum of $\sum_{n=0}^{4} 3n + 1$

5. (We do) What is the sum of the series $\sum_{n=1}^{5} 2n - 1$?

6. (They do) Use the calculator to determine the sum of the series $\sum_{n=1}^{20} n^3 - 10n^2$?

7. (They do) A student has taken three math tests so far this semester. His scores for the first three tests were 75, 79 and 83.
   a. Suppose his test scores continue to improve at the same rate. What will be his grade on the sixth and final test?

   b. What will be his total score for all six tests?

You do Practice 9-4: Complete your assignment on a separate sheet of paper. Show all work.

1. Use S_n to find the sum of each finite arithmetic series.
   a. 4 + 7 + 10 + 13 + 16 + 19 + 22
   b. 10 + 20 + 30 + … + 110 + 120

2. Write each arithmetic series in summation notation.
   a. 3 + 6 + 9 + 12 + 15 + 18 + 21
   b. 1 + 5 + 9 + …+ 41 + 45

3. What is the difference between an arithmetic sequence and an arithmetic series?

4. Is it possible to have more than one arithmetic series with four terms whose sum is 44? Explain.

5. A student writes the arithmetic series 3 + 8 + 13 +…+ 43 in summation notation as $\sum_{n=3}^{8}(3 + 5n)$. Describe and correct the error.

9-5 Geometric Series

Objective

Students will define, geometric series and find their sums.

State Standards

A2. F.BF. A.1a Write a function that describes a relationship between two quantities.
A2. F.BF.A.2 Know and write arithmetic and geometric sequences with an explicit formula and use them to model situations.
A2. F.LE.A.1 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a table, a description of a relationship, or input-output pairs**.**

Vocabulary

____- the sum of the terms of a geometric sequence.

An infinite series _____________________________ when | r | < 1. (It has a sum)

An infinite series _____________________________ when | r | ≥ 1. (does not have a sum)

take note

Key Concept Sum of a Finite Geometric Series

The sum S_n of a finite geometric series $a_1 + a_1 r + a_1 r^2 + \cdots + a_1 r^{n-1}, r \neq 1$, is

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

where a_1 is the first term, r is the common ratio, and n is the number of terms.

Examples

1. (I do) What is the sum of the finite geometric series?
   a. $3 + 6 + 12 + 24 + \ldots + 3072$; $n = 11$

   b. (I do) $\sum_{n=0}^{20} 4\left(\frac{1}{2}\right)^n$

2. (I do) Write the series using summation notation. Then evaluate.
   a. $4 + 12 + 36 + \ldots$; $n = 15$

3. (We do) Does the series converge or diverge? If it converges, what is the sum?
   a. $1 + \frac{1}{2} + \frac{1}{4} + \cdots$
   b. $\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)\left(-\frac{5}{4}\right)^{n}$

4. (We do) The height a ball bounces is less than the height of the previous bounce due to friction. The heights of the bounces form a geometric sequence. Suppose a ball is dropped from one meter and rebounds 95% of the height of the previous bounce. What is the total distance traveled by the ball when it comes to rest?

You do Practice 9-5: Complete your assignment on a separate sheet of paper. Show all work.

1. Use the sum formula to evaluate each finite geometric series.
   a. $\frac{1}{5} + \frac{1}{10} + \frac{1}{20} + \frac{1}{40} + \frac{1}{80}$
   b. $9 - 6 + 4 - \frac{8}{3} + \frac{16}{9}$

2. Write the series using summation notation. Then evaluate. $15 + 12 + 9.6 + \ldots$; $n = 15$

3. Determine whether each infinite geometric series diverges or converges.
   a. $1 - \frac{1}{6} + \frac{1}{36} - \frac{1}{216} + \cdots$
   b. $\frac{1}{64} + \frac{1}{32} + \frac{1}{16} + \cdots$

4. A classmate uses the formula for the sum of an infinite geometric series to evaluate $1 + 1.1 + 1.21 + 1.331 + \ldots$ and gets -10. What error did your classmate get?

5. Explain how you can determine whether an infinite geometric series has a sum.

6. How are the formulas for the sum of a finite arithmetic and finite geometric series similar? How are they different?